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THE PROBABILITY OF DETECTING COLOURED OBJECTS ON COLOURED BACKGROUNDS BASED ON A STATISTICAL MODEL OF THE THRESHOLD OF COLOUR VISION

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ABSTRACT

Using a statistical model of the threshold of colour vision, a new expression is obtained to calculate the detection probability of colour objects against colour backgrounds. The method shows that in the day sight area, this probability is completely determined by the individual criterion of decision making about an object's presence, by addition functions of the RGB colorimetric system, by radiance spectral concentrations of the object and background, and by the angular dimensions of the object.

Keywords: visual system of a person, theory of statistical solutions, optimum receiver, detection probability, direct and reverse problems

1. INTRODUCTION AND PROBLEM FORMULATION

The existing theories of colour vision focus on the explanation of the psychophysical aspects of colour perception [1], in particular on an observer's ability to equalise colours visually. Theories explaining the reasons for the existence of colour discrimination thresholds, which allow calculating the probability of detection of colour objects, were not identified in the existing literature.

Modern methods of calculating colour detection thresholds are based on an empirical approach, which does not explain why colour discrimination threshold arise.

Since the effectiveness of the application of the statistical solution theory to the calculation of the detection thresholds of monochrome images in [2–4] has been proved, it can be considered a natural generalisation of the statistical approach to the processes of detecting coloured objects, i.e. there is scope for developing a statistical theory of the threshold of colour sight.

As the statistical approach is based on the spectral sensitivity of radiation receivers, and the most appropriate theory of colour sight is three-component theory of Young-Helmholtz [1], the most sound approach is to develop a statistical theory based on the RGB physiological colorimetric system. This approach is both fundamental and practical. The International Commission on Illumination (CIE) recommends using the general colour rendering index [5] to evaluate the quality of colour rendition when developing new illumination devices. In accordance with [6, 7], calculations of a general colour rendition index are carried out in the CIE uniform-chromaticity-scale system 1964 U*V*W*. These R_a calculations use von Kries coefficients to exclude a systematic error arising due to the different colour adaptation of the visual system (VS) to the reference and examined light sources. These are also calculated in the RGB colorimetric system. The calculation of colour co-ordinates is performed according to the classical radiation expressions [6, 7]:

$$\Xi = \int_{380}^{780} L_{e\lambda}(\lambda)\bar{\xi}(\lambda)d\lambda\,, \qquad (1)$$

here $\Xi = \{R, G, B\}$, $L_{e\lambda}(\lambda)$ is radiance spectral concentration, λ is radiation wavelength, so the error in the definition of the addition functions $\bar{\xi}(\lambda)=\left\{\bar{r}(\lambda),\ \bar{g}(\lambda),\ \bar{b}(\lambda)\right\}$ inevitably affects the reliability of the obtained values of the general colour rendering index.

As has been shown in [8], classical methods for determining specific colour coordinates of the RGB colorimetric system, based on experiments with dichromate, lead to unpredictable errors. Therefore it becomes necessary to determine the curve for a subject with normal vision under natural eye adaptation conditions. The first step is to derive expressions for the detection probability of colour objects on colour backgrounds at specific RGB colour coordinates. Since such probabilities can be obtained through experimental studies of visual systems, solving this mathematically for a subject with normal colour vision can identify specific RGB colour coordinates.

This paper is devoted to solving the problem at hand and assumes that the known functions of the RGB colorimetric system are known.

2. METHOD OF SOLVING THE FORMULATED PROBLEM

Works [2–4], consider threshold characteristics of a human visual system in detail, based on a statistical approach. The main limitations of the methods they propose is that it can be only used for low levels of adaptation luminance and for observation of one-colour images, when only the rods apparatus of the visual system works. The first successful attempts to apply a statistical approach to explanation of colour thresholds of visual system were made in [9, 10], however this only addressed the detection of thresholds for monochromatic objects against white backgrounds. Obtaining calculation expressions for detection probability of arbitrary colour objects against colour backgrounds requires further study.

With regard to solving problems of single-colour image detection by the human visual system, a block diagram of the VS mathematical model is provided in [4, Fig. 1], based on an optimal statistical receiver [11]. A fundamental difference of the threshold colour vision mathematical model (TCVMM) from that given in [4] is the three receiver types (*R*, *G* and *B*) with reactions μ_{ri}, μ_{gi}, μ_{bi} and with a nonlinear dependence of these reactions on adaptation luminance.

According to the block diagram, a person's field of vision can have either an image of a coloured object on a colour background, or an image of a coloured background, differing only in chromaticity and luminance from the image of the object. Random output signals *R*, *G* and *B* of receptors (μ_{ri}, μ_{gi}, μ_{bi}) are transmitted via optic nerve fibres to the brain, which is represented in the block diagram by memory, analysis device (AD) and decision-making device (DMD), which decided whether an object is present or not in the visual field.

According to the theory of the optimum statistical receiver [4, 11], on which TCVMM is based, the analysis device (AD) calculates one-dimensional function of the credibility relation (Λ) equal to the relation of emergence probability (*P* [Y/S]) of random two-dimensional distributions of signals μ_{ri}, μ_{gi}, μ_{bi} (random *Y* implementation) at the output of radiation receivers matrix (RR) under the condition of emergence of the studied colour object against a colour background in the visual field, to emergence probability of (*P* [Y/0]) of the same *Y* implementation under the condition of colour background emergence without an object in the visual field.

$$\Lambda = \frac{p}{q}\frac{P[Y/S]}{P[Y/0]}, \qquad (2)$$

where *p* and *q* are priori probabilities of object and background emergence ($p + q = 1$).

The decision-making device (threshold device ThD) makes a decision on the presence of a colour object against a colour background in the visual field every time the calculated value Λ exceeds threshold value Λ_{th}. Otherwise, it makes the decision on the presence a background without an object in the visual field. The optimum receiver algorithm allows using the results of the previous work detecting one-colour objects [4] for analysing the threshold of colour sight as a mathematical model.

Using the results of [4], it is simple to obtain an expression to determine the logarithm of credibility relation (Z) in problems of calculating detection probability of a colour object against a colour backgrounds for the distribution law (*P*[*Z*]):

$$P[Z] = \frac{1}{2\pi\sigma_\Lambda}\exp\left(-\frac{(Z-m_\Lambda)^2}{2\sigma_\Lambda^2}\right), \qquad (3)$$

where $Z = \ln\Lambda = \sum_{i=1}^{N} \mu_i \ln\left(\frac{X_{oi}}{X_{\phi i}}\right) - \sum_{i=1}^{N} (X_{oi} - X_{\phi i})$,

μ_i is the totality of random output signals of the radiation receiver (RR): μ_{ri}, μ_{gi}, μ_{bi}; X_{oi}, $X_{\phi i}$ are mathematical expectations of matrix i RR output signal under the condition that a colour object is observed on a colour background and respectively that a colour background is observed without an object.

For statistically independent RRs, it is simple to obtain expressions [4] for mathematical expectation m_Λ and for dispersions σ_Λ^2 of the credibility relation logarithm provided that an object is in visual field:

$$m_\Lambda = \sum_{i=1}^{N} X_{oi} \ln\left(\frac{X_{oi}}{X_{\phi i}}\right) - \sum_{i=1}^{N} (X_{oi} - X_{\phi i}),$$
$$\sigma_\Lambda^2 = \sum_{i=1}^{N} X_{oi} \ln^2\left(\frac{X_{oi}}{X_{\phi i}}\right), \quad (4)$$

where N is number of RGB receivers in the RR matrix.

As the optimum receiver makes a decision on the presence of an object, if $\Lambda \geq \Lambda_п$, then taking into consideration the monotony of the logarithmic function provided that an object is within the visual field, the probability of correct detection is determined by the integration of the conventional distribution law ln(Λ) over values area ln(Λ) from ln($\Lambda_п$) to infinity:

$$P_{det} = \frac{1}{2\pi\sigma_\Lambda} \int_{\ln\Lambda_п}^{\infty} \exp\left(-\frac{(Z - m_\Lambda)^2}{2\sigma_\Lambda^2}\right) dZ = \Phi(y), \quad (5)$$

$$y = \frac{m_\Lambda - \ln\Lambda_ї}{\sigma_\Lambda}, \quad (6)$$

where $\Phi(y) = \frac{1}{2\pi} \int_{-\infty}^{y} e^{-t^2/2} dt$ is probability integral [12].

According to [13], with increasing luminance, the relation of "action current" pulse frequency at the receptor output of the visual system to the photon flux incident on the receptors decreases. Therefore, the effective transformation coefficient used in [4] for determining the receptor signal distribution law will be much less than one with daylight luminance. This makes it possible to use Poisson distribution law to descript receptor reactions in TCSMM as well. In this case, at $p = q$, the expression for credibility relation [2] will be as follows:

$$\Lambda = \prod_{i=1}^{N} \left(\frac{X_{oi}}{X_{эi}}\right)^{\mu_i} \exp\left(-(X_{oi} - X_{эi})\right). \quad (7)$$

As it was noted already, human colour vision is connected with presence of three types of receptors in the visual system with output signals μ_{ri}, μ_{gi}, μ_{bi} generating μ_i. Therefore, these signals should be explicitly added to the expression. With statistically independent (under the threshold conditions) RRs, we will group (similar to [10]) reactions of *R*, *G* and *B* receptors. Then expression (7) for the credibility relation will be as follows:

$$\Lambda = \prod_{i=1}^{n} \left(\frac{X_{окi}}{X_{фкi}}\right)^{\mu_{кi}} \left(\frac{X_{озi}}{X_{фзi}}\right)^{\mu_{зi}} \left(\frac{X_{осi}}{X_{фсi}}\right)^{\mu_{сi}}$$
$$\exp\begin{pmatrix} -(X_{окi} - X_{фкi}) - (X_{озi} - X_{фзi}) - \\ -(X_{осi} - X_{фсi}) \end{pmatrix}, \quad (8)$$

where n is the number of *RGB* triads in whole number N of model receivers, and $X_{окi}$, $X_{озi}$, $X_{осi}$, $X_{фкi}$, $X_{фзi}$, $X_{фсi}$ are conventional mathematical expectations of *R*, *G*, *B* receptors output signals in the presence of an object and background in visual field respectively.

After taking the logarithm of the left and right parts (8), we will obtain an expression for ln(Λ) as:

$$\ln(\Lambda) = \ln\left(\prod_{i=1}^{n} \Lambda_к \Lambda_з \Lambda_с\right) =$$
$$= \ln(\Lambda_к) + \ln(\Lambda_з) + \ln(\Lambda_с), \quad (9)$$

where Λ_r, Λ_g, Λ_b are particular credibility relations calculated for reactions of *R*, *G*, *B* receptors;

$$\Lambda_\xi = \prod_{i=1}^{n} \left(\frac{X_{o\xi i}}{X_{э\xi i}}\right)^{\mu_{к\xi i}}$$
$$\exp\left(-(X_{o\xi i} - X_{э\xi i})\right), \quad \xi = \{r, g, b\}. \quad (10)$$

Thus, the detection probability of an object by chromaticity or luminance against a colour background will be determined by expressions (5) – (6), in which parameters of the distribution law ln(Λ) are specified by expressions:

$$m_{\Lambda} = m_{\Lambda r} + m_{\Lambda g} + m_{\Lambda b}, \quad \sigma^2_{\Lambda} = \sigma^2_{\Lambda r} + \sigma^2_{\Lambda g} + \sigma^2_{\Lambda b}, \qquad (11)$$

where

$$m_{\Lambda\xi} = \sum_{i=1}^{N} X_{o\xi i} \ln\left(\frac{X_{o\xi i}}{X_{\phi\xi i}}\right) - \sum_{i=1}^{N} (X_{o\xi i} - X_{\phi\xi i}),$$
$$\sigma^2_{\Lambda\xi} = \sum_{i=1}^{N} X_{o\xi i} \ln^2\left(\frac{X_{o\xi i}}{X_{\phi\xi i}}\right), \quad \xi = \{r, g, b\}. \qquad (12)$$

3. DETERMINATION OF TCVMM RECEPTOR REACTION DEPENDENCE ON THE OBSERVED LUMINANCE

To derive an expression for the detection probability of colour objects, we need to find the mathematical model receiver output reaction dependence on the visionable luminance. This is not straight forwards, as the receptor is nonlinear within the day sight area ($L_v \geq 10$ cd/m^2), i.e. with a nonlinear dependence of the "action current" pulse frequency in the fibres of the optic nerve [13] on photon radiation flux. This leads to the fact that the photometric contrast of the observed objects differs from the signal contrast of the mathematical model RR output. In [3, 10] an RR reaction's dependence on the sought luminance is presented, however it is obtained for only one type of receptor, which is insufficient for a colour sight model.

One of the basic colorimetric principles asserts that in daytime vision the chromaticity of permanent spectral composition objects does not depend on the changing luminance of the objects [7]. According to expression (1) reactions of *R*, *G* and *B* receptors ($X_{к}, X_{з}, X_{с}$) are proportional to colour co-ordinates of the *RGB* physiological colorimetric system. It follows from this:

$$\frac{X_r}{X_r + X_g + X_b} = \xi \rightarrow X_{\xi} = \xi X, \quad \xi = \{r, g, b\}, \qquad (13)$$

where $X = X_r + X_g + X_b$ is the sum of *R*, *G*, *B* receptor reactions depending on background luminance, and *r*, *g*, *b* are colour co-ordinates independent of luminance.

Using expression (13) one can determine the contrast connection at the receptor $KR = \Delta X/X$ output with photometric contrast $K = \Delta L_v/L_v$:

$$K_R = \frac{X_{\hat{\imath}\hat{e}} + X_{\hat{\imath}\c{c}} + X_{\hat{\imath}\tilde{n}} - X_{\hat{o}\hat{e}} - X_{\hat{o}\c{c}} - X_{\hat{o}\tilde{n}}}{X_{\hat{o}\hat{e}} + X_{\hat{o}\c{c}} + X_{\hat{o}\tilde{n}}}, \qquad (14)$$

where X_{or}, X_{og}, X_{ob} and $X_{\phi r}, X_{\phi g}, X_{\phi b}$ are mathematical expectations of output reactions of any *R*, *G* and *B* receptor sought an area within object contour under the condition of observing a colour object against a background and colour background only, respectively.

With due regard for (14) it is simple to derive:

$$K_R = \frac{X_o - X_{\phi}}{X_{\phi}}, \qquad (15)$$

where X_o, X_{ϕ} are sums of mathematical expectations of receiver signals under the condition of sighting of an object and background respectively.

Standard objects, which are used in colorimetry, have angular dimensions from two to ten degrees. With such angular dimensions and with an adaptive luminance greater than 10 cd/m^2, values of threshold luminance contrasts on any background and any object chromaticity, as well as signal contrast values at the receptor output, are much less than one [7].

This allows limiting by two terms of Taylor's series expansion of function $X_{\hat{\imath}}(L)$ [12] in expression (15) at vicinity $X=X_{\phi}$. Then:

$$K_R = \frac{dX}{dL_v}\frac{\Delta L_v}{X} = \frac{dX}{dL_v}\frac{L_v}{X} K. \qquad (16)$$

Use of this expression in TCVMM makes it possible to derive a differential equation for $X\ (L_v)$ dependence. Let's now transform expressions (12) for m_{Λ} and σ^2_{Λ} as well, having used difference of the receiver signals (ΔX_i) sighting object and background:

$$m_{\Lambda} = \sum_{i=1}^{n} (X_i + \Delta X_i) \ln\left(1 + \frac{\Delta X_i}{X_i}\right) - \sum_{i=1}^{n} \Delta X_i,$$
$$\sigma^2_{\Lambda} = \sum_{i=1}^{n} (X_i + \Delta X_i) \ln^2\left(\frac{X_i + \Delta X_i}{X_i}\right). \qquad (17)$$

Having expanded logarithm in (17) into Taylor's series in vicinity of one, we will be limited by two expansion terms and consider a frequent case of observing equally-bright objects against a uniform background. Under these conditions, out of object contour $\Delta Xi = 0$ and within it, the difference of receiver signals has a constant value ΔX.

Within the object contour, X_i doesn't depend on the receiver number as well and is equal to X. Therefore, expressions (17) become simpler:

$$m_\Lambda = \frac{1}{2}\sum_{i=1}^{n}\frac{(\Delta X_i)^2}{X_i} = \frac{1}{2}t\frac{(\Delta X)^2}{X},$$
$$\sigma_\Lambda^2 = \sum_{i=1}^{n}\frac{(\Delta X_i)^2}{X_i} = t\frac{(\Delta X)^2}{X}, \quad (18)$$

where t is number of RRs sighting a space area within the object contour.

With little differences ΔX typical for threshold contrasts of big angular size objects, it can be expressed by a derivative of searched dependence of receiver reaction X of background luminance L_v in the form of $\Delta X = \Delta L_v \, dX/dL_v$, where ΔL_v is object – background luminance difference. Then expression (6) for probability integral argument determining probability of object detection will be as follows:

$$y = \frac{m_\Lambda - \ln \Lambda_п}{\sqrt{2m_\Lambda}}. \quad (19)$$

According to experimental studies of luminance threshold contrasts [7], with angular object dimensions greater than 2º and background luminance greater than 10 cd/m^2, Weber-Fechner law is valid, i.e. when the background luminance changes, threshold contrast remains constant. This means that detection probability also does not depend on background luminance, so derivative $dy/dL_v = 0$. As under the threshold conditions ($P_{ob} = 0.5$) argument of the probability integral is equal to zero [12], then from expression (19) a differential equation for X dependence on luminance L_v can be obtained:

$$\frac{t}{2}\frac{d}{dL_v}\left[K^2\left(\frac{dX}{dL_v}L_v\right)^2 \bigg/ X\right] = 0. \quad (20)$$

As photometric contrast $K = \Delta L_v / L_v$ in the Weber-Fechner area does not depend on luminance, solution (20) is a function of luminance square logarithm:

$$X = C_1 \ln^2(C_2 L_v), \quad (21)$$

where C_1 *and* C_2 are arbitrary integration constants not dependent on luminance.

With due regard for (13), the obtained result allows drawing the conclusion that nonlinearity of all three types of cones is identical and is determined by expression (21) with different C_1 values and coinciding C_2 values.

4. CALCULATION RATIO FOR DETECTION PROBABILITY OF A COLOUR OBJECT AGAINST A COLOUR BACKGROUND

Detection probability of a colour object against a colour background is determined by expressions (5) and (6). The expression for probability integral argument (6) can be transformed in accordance with (11) and (18) as follows:

$$y = \frac{\sum_\xi \Pi_\xi - \ln \Lambda_п}{\sqrt{2\sum_\xi \Pi_\xi}},$$
$$\Pi_\xi = \frac{t\left(\Delta L_\xi \, dX_\xi / dL_\xi\right)^2}{2X_\xi}, \ \xi = \{r, g, b\}. \quad (22)$$

Taking into account the obtained dependence of RR reactions on luminance (21), components Π_j of expression (22) will be as follows:

$$\Pi_\xi = 2tC_{1\xi}\left(\frac{\Delta L_\xi}{L_\xi}\right)^2, \quad (23)$$

where according to [7],

$$L_\xi = 683\beta_\xi \int_{380}^{780} L_{e\lambda}(\lambda)\bar{\xi}(\lambda)d\lambda, \quad (24)$$

$$\Delta L_\xi = 683\beta_\xi \int_{380}^{780} \left(L_{e\lambda o}(\lambda) - L_{e\lambda}(\lambda)\right)\bar{\xi}(\lambda)d\lambda, \quad (25)$$

where $L_{e\lambda o}(\lambda)$ object, $L_{e\lambda}(\lambda)$ is radiance spectral concentration of the background, β_ξ are luminance coefficients of RGB colorimetric system, ξ = {*r*, *g*, *b*}, $\bar{\xi}(\lambda) = \{\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)\}$, C_{1r}, C_{1g}, C_{1b} are constants not dependent either on the object and background spectrum or on the object angular size.

If to introduce solid angles Ω of the object and of instant visual field ω of any RR, then the number of R, G, B triads of the receivers in RR matrix sighting an area within the object contour will be determined by the following expression:

$$t = \frac{\Omega}{3\omega}. \qquad (26)$$

Formulas (22) – (26) completely determine calculation expression for *y*.

5. CONCLUSIONS

Applying the optimum statistical receiver theory to the detection processes of a colour object against a colour background allowed obtaining a calculation expression for the probability of its detection which showed the following:

1. Luminance coefficients of primary colours of the RGB colorimetric system do not influence the detection probability of colour objects against colour backgrounds. When detecting a colour object, the influence of addition functions of the RGB colorimetric system is principal, and maximum values of addition functions also do not influence the detection probability.
2. In case addition functions are known, the detection probability of colour objects on a colour background is unambiguously determined by value $\Lambda_{п}$, by spectral luminance distributions over surfaces of the object and background and by angular dimensions of the object.
3. Constant coefficients of the mathematical model, including a criterion for decision making by a person (the criterion is determined by $\ln(\Lambda_{п})$ value), do not depend on the angular size of the object and on the spectral luminance distribution over the object and background and can be determined when normalising the mathematical model.
4. The obtained expression allows calculating the detection probability of colour objects on colour backgrounds with arbitrary spectra (chromaticities) of the objects and backgrounds.
5. However, to solve a reverse problem of finding $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ values experimentally studying the detection probability of colour objects using expressions (5) and (22) – (26), the development of a special protocol and criteria for the experimental installation are required.

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