Light & Engineering
Vol. 28, No. 6, pp. 95–104, 2020

https://doi.org/10.33383/2019-098
Svetotekhnika # 3, 2020, pp.78–85

METHODS OF INCREASING SPECTRAL RESOLUTION OF IMAGING SPECTROMETERS BUILT ON THE BASIS OF MULTI–CHANNEL RADIATION DETECTORS

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ABSTRACT

The article proposes the methods of object shooting by means of a spectrometer based on a multi-channel radiation detector and further processing of its results allowing spectral resolution of such spectrometers significantly to increase with the same original spatial resolution. The mathematical model of the shooting process is provided. It is determined that restoration of spectral radiance of objects based on the shooting data using the proposed method is a mathematically incorrect inverse task. The Greville method, the method of wavelet transformation, the Tikhonov regularisation method, and the Godunov method were considered as methods for its solution. The results of computational modelling of the considered methods are shown and it is found that restoration of spectral radiance of objects based on the shooting data using the considered methods is possible and relative error of restoration is at a fraction of per cent scale. It is determined that the wavelet transformation method is an optimal method of solution of the incorrect spectral radiance restoration task. It is also shown that the proposed method of imaging spectrometry is applicable both when using matrix radiation detectors with increased number of narrow-band filters and when using widely spread standard three-channel matrix *RGB* detectors of radiation.

Keywords: imaging spectrometry, incorrect tasks, multi-channel shooting, spectrum restoration, optical filters

1. INTRODUCTION

Imaging spectrometers are rather actively used in science and industry. The key distinction of imaging spectrometers is the capability to record spectral characteristics of each point of a two-dimensional image of an object. For conventional representation of a data array acquired when measuring by means of an imaging spectrometer, the data cube concept is used [1]. The data cube is a three-dimensional structure formed by spectral characteristics of radiation reflected from the studied surface on one plane and by corresponding spatial coordinates on the two other planes.

In terms of the number of spectral channels used for measurement and spectral resolution, imaging spectrometers are conventionally identified as multispectral and hyperspectral ones. The multispectral instruments are the ones registering radiation in 4 to 100 spectral channels with resolution less than 10 nm [2, 3]. The instruments with high spectral resolution have the best specifications among hyperspectral instruments [4, 5]. They have over 1000 spectral channels and their resolution is better than 1 nm. In order to reach high resolution, these hyperspectrometers are equipped with dispersing systems of various types or built based on the principle of a Fourier spectrometer [6–9]. Due to the said distinctions and high requirements to the elements of the structures of dispersing and interference systems, these instruments have large dimensions, low operational characteristics and high cost [10–11]. Moreover, for processing of information and formation of the data cube, it is necessary to provide a mechanism of scanning in one of the coordinates of the cube: either spatial or spectral one.

Imaging spectrometers with 100 up to 1000 channels and resolution of 1 nm to 10 nm are classified as medium-resolution spectrometers.

Multispectral instruments are the simplest imaging spectrometers with the best operational characteristics [12, 13]. They comprise a combination of a lens and a sensing matrix installed on the image plane. The sensors (micro pixels) are equipped with narrow-band selective filters with their positioning on the matrix defining the type of the matrix radiation detector (RD): band, tile or mosaic-type. In terms of construction of imaging spectrometers, mosaic-type matrix RD's are preferred with groups of micro pixels with selective filters united into one macro pixel, i.e. the same principle is used as in standard *RGB* matrices, however, the number of spectral channels is increased from 3 to 4, 8, 16, etc. Increase of spectral resolution of such instrument is reached by increasing the number of channels in a macro pixel, but it increases the dimensions of a macro pixel and, therefore, reduces spatial resolution [14]. The method under consideration allows us to increase spectral resolution of a multispectral instrument up to the level common for a medium-resolution hyperspectrometer with the same spatial resolution.

The method is based on shooting of a studied object through several filters with known spectral characteristics of transmission and then the parameters of the obtained images are processed mathematically to find spectral characteristics of the object within each macro pixel with spectral resolution significantly exceeding capabilities of a multispectrometer. The article also demonstrates that not only multispectral matrix RDs operating in four or more channels can serve as a basis for an imaging spectrometer utilising the proposed method but also standard three-channel *RGB* RDs can, and final spectral resolution can reach several nanometres.

2. THE METHOD

Determination of spectral characteristics of continuous, uniformly illuminated and diffusively reflecting objects using data from imaging spectrometers based on multispectral RDs is a problem of interpretation of the input relation using results of indirect measurements [15, 16] such as response of RD to an input signal [17]. The article analyses reconstruction of a spectral characteristic, namely spectral radiance (SR) $L_e(\lambda)$ of the objects, based on the signal quantity U from pixels of the multispectral RD using known functions of spectral responsivity of corresponding channels $S(\lambda)$. Quantities of signals from pixels of a matrix RD within a single macro pixel are written as linear integral equations:

$$\begin{cases} \int_{\lambda_1}^{\lambda_2} S_1(\lambda)\cdot L_e(\lambda)d\lambda = C\cdot U_1, \\ \int_{\lambda_1}^{\lambda_2} S_2(\lambda)\cdot L_e(\lambda)d\lambda = C\cdot U_2, \\ \quad\vdots \qquad\qquad = \qquad \vdots \\ \int_{\lambda_1}^{\lambda_2} S_m(\lambda)\cdot L_e(\lambda)d\lambda = C\cdot U_m, \end{cases}$$

where U_1, U_2, U_m are the signals values of a pixels corresponding to a specific channel obtained when shooting an object by means of a camera; $S_1(\lambda)$, $S_2(\lambda)$,...$S_m(\lambda)$ are the functions of spectral responsivity of a matrix RD pixel in each channel; λ_1 и λ_2 are the boundaries of operating range of wavelengths; m is the number of channels; C is the ratio not depending on λ and defined by parameters of equipment and conditions of shooting (diameter of the lens entrance pupil, distance to the object, field of view, etc.).

For approximate calculations of corresponding integrals with constant increment $\Delta\lambda$, the following system of equations is used

$$\begin{cases} \sum_{i=1}^{n} L_e(\lambda_i)\cdot S_1(\lambda_i)\cdot v(\lambda_i) = C\cdot U_{1\tau j}, \\ \sum_{i=1}^{n} L_e(\lambda_i)\cdot S_2(\lambda_i)\cdot v(\lambda_i) = C\cdot U_{2\tau j}, \\ \quad\vdots \qquad\qquad = \qquad \vdots \\ \sum_{i=1}^{n} L_e(\lambda_i)\cdot S_m(\lambda_i)\cdot v(\lambda_i) = C\cdot U_{m\tau j}, \end{cases} \tag{1}$$

where $L_e(\lambda_i)$ are the values of SR of the object at discrete points of the operating region of the spectrum; $S_1(\lambda_i)$, $S_2(\lambda_i)$, ... $S_m(\lambda_i)$ are the discrete values of the pixel spectral responsivity with narrow-band filter of the matrix RD; $v(\lambda_i)$ is the weighting factors of the increment depending on the method of numerical integration: triangular method, trapezoidal method, the Simpson method, etc.; U_1, U_2,... U_m are the signal quantities of pixels acquired when shooting the object by means of a multispectral camera; m is the number of narrow-band filters; n is the number of split points over the spectrum; i is the split step number.

With the known functions $S_1(\lambda)$, $S_2(\lambda)$, $S_m(\lambda)$ and values U_1, U_2, U_m, the system (1) resolves itself to a system of linear equations in the set of selected values $L(\lambda_i)$. With that, the number of values $L_e(\lambda_i)$ is defined by the value of n. The error of these values is mainly defined by the constructive matrix of equations of the system (2) and the nature of changes of the function $L(\lambda)$. Usually, the system (1) does not allow to recover information on the function $L_e(\lambda)$ with required accuracy, therefore, in order to obtain additional information on SR of the object, it is suggested to make several additional shots through special optical filters with known spectral transmission functions $\tau_j(\lambda)$ (j = 1, 2...p). The system of equations (1) is also supplemented by $m \times p$ more similar equations:

$$\begin{cases} \sum_{i=1}^{n} L_e(\lambda_i)\cdot S_1(\lambda_i)\cdot v(\lambda_i)\cdot \tau_j(\lambda_i) = C\cdot U_{1\tau j}, \\ \sum_{i=1}^{n} L_e(\lambda_i)\cdot S_2(\lambda_i)\cdot v(\lambda_i)\cdot \tau_j(\lambda_i) = \\ \quad = C\cdot U_{2\tau j},\ j=1...p \\ \quad \vdots \qquad = \qquad \vdots \\ \sum_{i=1}^{n} L_e(\lambda_i)\cdot S_m(\lambda_i)\cdot v(\lambda_i)\cdot \tau_j(\lambda_i) = C\cdot U_{m\tau j}, \end{cases} \tag{1+}$$

where $U_{1\tau j}$, $U_{2\tau j}$, … $U_{m\tau j}$ are signal quantities of the pixels acquired when shooting the object by means of a camera with corresponding narrow-band filters through a filter with transmission function $\tau_j(\lambda_i)$. The system of equations (1+) may be enhanced by including the system (1) in it. For this purpose, it is necessary to introduce the value of the parameter j = 0 in the system (1+) and to take $\tau_0(\lambda_i)$ = 1, i.e. to consider that the object is being shot without a filter.

Selection of the required number of filters and optimal nature of their transmission functions is defined by necessary accuracy of determination of the function $L_e(\lambda_i)$, required dynamic characteristics of the image processing system and other parameters of the design specification.

For modelling and computational solution, it is convenient to write the system of equations (1+) in the matrix form:

$$S\cdot \boldsymbol{l} = \boldsymbol{u}, \tag{2}$$

where S is a $m\cdot(p+1)\cdot n$-order matrix taking into account spectral responsivity of the pixels of the matrix RD, known spectral functions of transmission $\tau_j(\lambda_i)$ and weighting factors of increment $v(\lambda_i)$; $\boldsymbol{l}$ is the vector consisting of n elements defining SR of the object; $\boldsymbol{u}$ is the vector consisting of $m\cdot(p+1)$ defining the signal quantities of pixels acquired when shooting the object by means of a multispectral camera itself and with additional filters.

2.1. The Greville Method

With high requirements to spectral resolution of an imaging spectrometer and natural strive to their technical implementation, the amount of unknown values of $L_e(\lambda_i)$ turns out to be large and sometimes significantly exceeds the number of equations (1+).

With that, the problem (2) may either have no solution or have a non-unique solution [18–20], i.e. it may be set mathematically incorrectly, therefore, a pseudo solution is used in such cases [21–23], i.e. such vector $\boldsymbol{l}$ that measures up the following functional [24]:

$$\left\| S\cdot \boldsymbol{l} - \boldsymbol{u} \right\|^2 \to min.$$

On the other hand, in consequence of the known theorem on perpendicular, the normal pseudo solution is determined with a single value and may be found using the equation

$$\boldsymbol{l} = S^{+}\cdot \boldsymbol{u}, \tag{3}$$

where S^+ is a pseudo-inverse matrix obtained by means of pseudo-inversion of the matrix S.

Pseudo-inversion may be understood as solution of the problem of the best approximation using the least squares method. The pseudo-solution method is one of the simplest ways to restore spectral radiance of objects using the data of multichannel shooting, and the pseudo-inverse matrix was found using the Greville method in this work.

2.2. The Wavelet Transformation Method

The equation (2) is an inverse incorrect problem [25, 26], and its solution written as (3) is a simple one but has a grave disadvantage. It resides in the fact that the distinctions of the problem discussed in this article often make the matrix S ill-conditioned (due to linear dependence between the rows of the matrix and therefore low stability of the solution to errors of the right-hand side of the equation (2)). In order to eliminate this factor of accuracy reduction of original luminance spectre reconstruction, wavelet transformation of the function may be used [27, 28].

Wavelet transformation of a regular signal is its representation in the form of a generalised series or a Fourier integral over the system of basic functions constructed from the mother (original) wavelet $\psi(\lambda)$ with specific properties gained using operations of time shift b and change of time scale a. For the set values of parameters a and b the function $\psi_{ab}(\lambda)$ is the wavelet generated by the mother wavelet $\psi(\lambda)$ [29]. Wavelets are used for shortening excessive information in this article. Each row of the matrix S is expanded in the basis and then not the elements of the matrix S, the number of which is defined by the amount of points of split over the spectrum and may reach tens and hundreds, are used in the solution of the equation (2) relative to $\boldsymbol{l}$ but the wavelet coefficients of its expansion in the basis the number of which is defined by the number of functions included in the basis, which ultimately reduces the condition number μ by orders of magnitude. The erf integral $\psi(\lambda)$ = erf(λ) was used as a mother function of wavelets in this article and the expansion basis consisted of 8 functions (i.e. k = 8) and was written in the following form:

$$\begin{cases} \psi_0(x) = 1, \\ \psi_1(x) = \mathrm{erf}(2 \cdot x), \\ \psi_2(x) = \mathrm{erf}\left(4 \cdot (x - \frac{1}{2})\right), \;\; \psi_5(x) = \mathrm{erf}\left(8 \cdot (x + \frac{1}{2})\right), \\ \psi_3(x) = \mathrm{erf}\left(4 \cdot (x + \frac{1}{2})\right), \;\; \psi_6(x) = \mathrm{erf}\left(8 \cdot (x - \frac{3}{4})\right), \\ \psi_4(x) = \mathrm{erf}\left(8 \cdot (x - \frac{1}{4})\right), \;\; \psi_7(x) = \mathrm{erf}\left(8 \cdot (x + \frac{3}{4})\right), \end{cases} \quad (4)$$

where x is a variable representing wavelength λ normalised to the standard interval [–1; 1] similar to [30].

SR of the object with use of wavelet transformation of the matrix S was defined using the following formula acquired from the expression (3)

$$\boldsymbol{l} = \Phi \cdot (S \cdot \Phi)^{+} \cdot \boldsymbol{u},$$

where Φ is the matrix of the values of basic functions from the expressions (4) with order of ($k \times n$); $(S \cdot \Phi)^{+}$ is the matrix acquired by pseudo inversion of the matrix $S \cdot \Phi$.

2.3. The Tikhonov Regularisation Method

Since it is impossible to obtain an accurate solution of the equation (2) stable to minor changes of input data due to the incorrect nature of the problem of radiance spectrum reconstruction, it is necessary to search for some approximated solution [31]. The equation (2) being solved is an operator equation of the first kind [32]; in [25; 33], it is shown that its solution is equivalent to the solution of the problem of functional minimisation:

$$M^{\alpha}(\boldsymbol{l},\boldsymbol{u}) \equiv \|S \cdot \boldsymbol{l} - \boldsymbol{u}\|^2 + \alpha \cdot \|\boldsymbol{l}\|^2 \to \min, \quad (5)$$

$$l \in L, \quad \alpha > 0,$$

where $M^{\alpha}(\boldsymbol{l}, \boldsymbol{u})$ is the Tikhonov smoothing functional, α is the regularisation parameter.

The regularised solution of the problem (2) is determined as the only solution of the Euler equation [25, 31]

$$(S' \cdot S + \alpha \cdot E) \cdot \boldsymbol{l} = S' \cdot \boldsymbol{u} + \alpha \cdot \boldsymbol{u}^{*},$$

where E is the unity matrix of the n degree.

The canon form of the regularisation method which was used in this article corresponds to the case $\boldsymbol{u}^{*} = 0$.

When practically applying this method, the algorithmic methods of α parameter selection are very important. One of them is based on the paper [34] and comprises selection of α using the values of the functional $M^{\alpha}(\boldsymbol{l}, \boldsymbol{u})$ on the regularised solutions from the constraint (5). The solution of such problem is found based on the principle of generalised residual [35] using Newton step-by-step approximation on the α grid:

$$\alpha_{s+1} = \alpha_s + \beta; \; s = 0, 1, 2\ldots; \; \beta = 0.001.$$

2.4. The Godunov Method

According to [36, 37], application of additional information about the fact that the desired solution has not too large second derivatives allows us to supplement it in such a way when compiling the system of linear equation, that such supplementation significantly lowers the condition number and makes the system solvable. In such case, the system of equations (1+) is written as the system

$$\begin{bmatrix}(1-\tau)\cdot S\\ \tau\cdot B\end{bmatrix}\cdot \boldsymbol{l}=\begin{bmatrix}(1-\tau)\cdot \boldsymbol{u}\\ 0\end{bmatrix}, \tag{6}$$

where B is the matrix with n order with its elements (b_{ij}) being such that

$$b_{ij}=\frac{-2}{(1/n-1)^2}\text{ при } i=j;$$

$$b_{ij}=\frac{-1}{(1/n-1)^2}\text{ при } i=j-1 \text{ и } i=j+1;$$

$$b_{ij}=0 \text{ при } |i-j|\geq 2;$$

τ is the ratio defined from the expression $\tau = 1/(n-1)^2$.

It was expected that, provided the above conditions of calculations of the elements of the matrix B and the ratio τ are complied with, the normal solution of the system (6) which may be written as

$$((1-\tau)^2\cdot S'\cdot S+\tau^2\cdot B'\cdot B)\cdot \boldsymbol{l}=(1-\tau)^2\cdot S'\cdot \boldsymbol{u},$$

would be proximate to the vector we are interested in [24].

2.5. The Modelling Procedure

The described method of determination of the object SR using the data of indirect measurements was checked using computational modelling in accordance with the expression (1+). Uniform diffusely scattering coloured plates were used as test samples. The SR curves of these samples, $L_{e1}(\lambda_i)$, $L_{e2}(\lambda_i)$ and $L_{e3}(\lambda_i)$, were determined by means of a certified device: spectrophotometer *Perkin Elmer Lambda* 950 (Fig. 1).

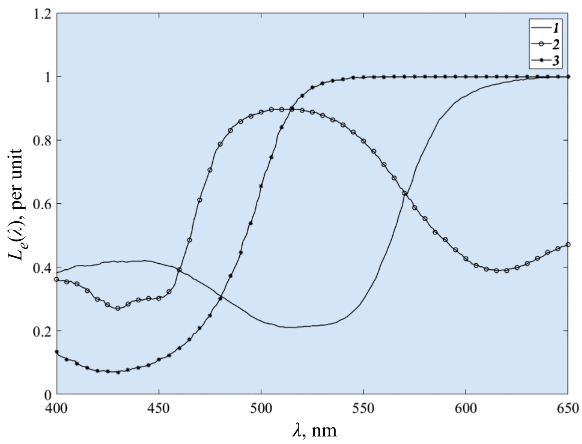

Fig. 1. Relative SR curves of samples 1 (*1*), 2 (*2*) and 3 (*3*)

Each sample was modelled separately. Multichannel mosaic-type matrix RD's are manufactured with macro pixels consisting of some fixed number of pixels with specific filter (i.e. the number of channels is m). Two RDs were considered in the course of modelling: a standard *RGB* matrix with three channels (m = 3) and a multispectral eight-channel mosaic-type matrix RD with its macro pixels consisting of eight pixels with narrow-band filters (m = 8). Computational modelling was conducted separately for each matrix. The curves of spectral responsivity of the *RGB* matrix, $S_R(\lambda)$, $S_G(\lambda)$ and $S_B(\lambda)$, and of the multispectral eight-channel matrix, $S_1(\lambda)$, $S_2(\lambda)$, …, $S_8(\lambda)$, are shown in Fig. 2, *a* and *b*, respectively.

Colour glasses ZhZS-5 (ЖЗС-5), ZhZS-18 (ЖЗС-18), SZS-16 (СЗС-16) and SS-1 (СС-1) were selected from the optic glass catalogue and used as optic filters in the course of computational modelling; their transmission functions, $\tau_1(\lambda_i)$, $\tau_2(\lambda_i)$, $\tau_3(\lambda_i)$, and $\tau_4(\lambda_i)$ (p = 4), complied with Standard 9411–91. The said colour glasses were selected using the classic method comprising of selection of filters with maximum difference between the shapes of transmission curves and uniform covering of the entire operational range of λ [38].

At the first stage of modelling, the signal quantities of the pixels of the standard *RGB* matrix $U_R\,\tau_j$, $U_G\,\tau_j$, $U_B\,\tau_j$, (j = 0,…p) are determined from the direct solution of the equation (1+) using the known parameters, namely spectral responsivity of the pixels of the matrix RD with Bayer filters $S_R(\lambda_i)$, $S_G(\lambda_i)$, $S_B(\lambda_i)$, SR of the samples $L_e(\lambda_i)$ and transmission functions $\tau_j(\lambda_i)$ of additional p filters (j = 1, 2,…p). The signal quantities of the pixels of the standard *RGB* matrix are defined for each sample, i.e. for $L_{e1}(\lambda_i)$, $L_{e2}(\lambda_i)$, $L_{e1}(\lambda_i)$. The corresponding values of signal quantity of the pixels of the multispectral RD are obtained in the similar way. In such case, another quantity p of additional optical filters is used. To provide sufficient accuracy of modelling, the increment of split over spectrum was 1 nm for all functions, which corresponded to the total number of points of n = 251 for the spectral region of (400–650) nm.

The second stage of modelling comprised determination of SR, $L_{e1}^*(\lambda_i)$, $L_{e2}^*(\lambda_i)$ and $L_{e3}^*(\lambda_i)$, of each sample at n points using one of the above-men-

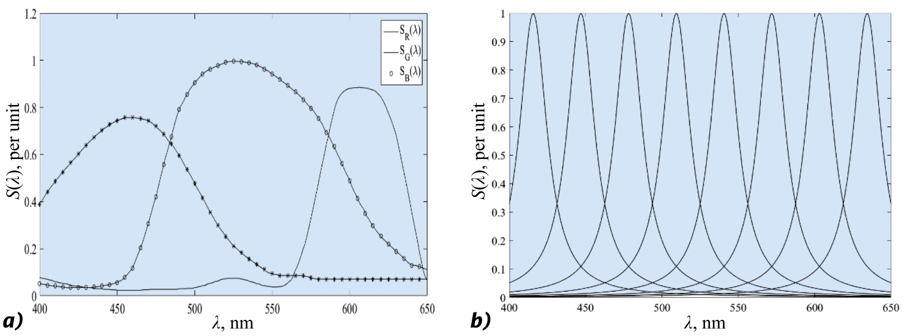

Fig. 2. Relative SR curves of pixels with Bayer filters of the three-channel *RGB* matrix (*a*) and the eight-channel multi-spectral matrix (*b*)

tioned methods of solution of the inverse incorrect task (2). Moreover, SR of the samples was defined separately for the two variants of modelling: with three-channel (m = 3) and eight-channel (m = 8) matrix RDs. Relative error expressed in per cent was the measure of concordance with the spectrum reconstructed in such a way.

3. RESULTS

The main results of mathematical modelling of the methods to determine the sample SR are presented below.

1. It appears to be impossible to solve the inverse incorrect task of determination of the object SR with accuracy specific for a medium-resolution spectrometer without using additional optical filters by means of any of the presented methods. Relative errors of the reconstructed and original SR of the samples are equal to tens of per cent. This is applicable to both multi-channel and *RGB* RDs.

2. Utilisation of additional filters for shooting of the object allows us to increase accuracy of SR definition with required spectral resolution. In most cases, one or two additional filters are sufficient for multispectral systems depending on the number of channels. It is appropriate to use the value p = 2 for the eight-channel radiation detector used for calculation in this article. It is found that the method of imaging spectrometry based on multi-channel RDs yields good results not only for matrix RDs with the number of channels increased as compared to the standard one, which was expected [39], but also for widely used three-channel matrix RDs. In the latter case, the number of optical filters involved in multi-channel shooting (p) is increased but remains within technically feasible limits and appears to be justified [40, 41]. The value p = 4 is assumed in this article.

3. Fig. 3 illustrates the results of reconstruction of the original SR using the Greville method, wavelet transformation, the Godunov method, and Tikhonov regularisation. The graphs demonstrate that all considered methods of solution of the equation (2) appear to be applicable for definition of SR of different samples. At the boundaries of the spectral region, almost all methods demonstrate some deviation between the original and the reconstructed curves, and this aspect shall be taken into account when using the methods in practice. The main challenge when using the Tikhonov method is calculation of the optimal value of the regularisation parameter α. In the course of modelling, the values of this parameter were calculated using the Newton step-by-step approximation method. They were equal to 0.0616 at m = 3 and 0.1481 at m = 8. The wavelet transformation method provides a more single-valued representation of the shape of the SR curve; therefore, in the problems requiring further integration of the values at specific points of the reconstructed curves in the calculations, it is preferable to use the method of wavelet transformation due to its less relative error of SR reconstruction.

4. The Table contains the values of relative error of restoration of SR of three objects using the above described methods of solution of incorrect inverse task. The Table makes it clear that computational modelling of the discussed method of multi-channel imaging spectroscopy and the methods of further data processing yield positive results and the original SR of the samples is reconstructed with low error. It may seem that the results listed in the Table are too optimistic but it should be noted that we are talking about just the errors of reconstruction of spectral curves in terms of the methods of solution of inverse problems. When using the Greville and the Godunov methods, deterioration of the results with increase of the number of matrix channels is observed, which is explained by high sen-

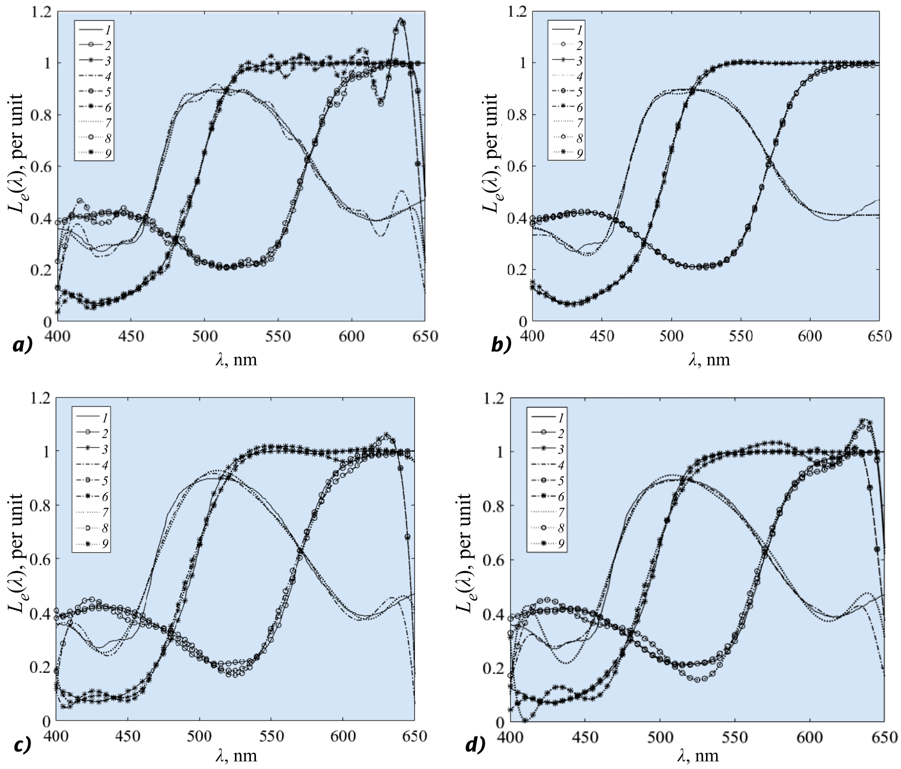

Fig. 3. Relative SR curves: original and reconstructed using the Greville method (*a*), wavelet transformation (*b*), Tikhonov regularisation (*c*) and the Godunov method (*d*). Original sample curves – *1*, *2*, *3*; the curves reconstructed using the three-channel shooting data (m = 3, p = 4) – *4*, *5*, *6*; the curves reconstructed using the data of eight-channel shooting (m = 8, p = 2) – *7*, *8*, *9*

sitivity of these methods to ill-conditioning of the matrix S included in the calculation; however, the Godunov method demonstrates more preferable results as compared to the Greville method. On the other hand, the methods of wavelet transformation and Tikhonov regularisation are applicable better with high condition number of the matrix S, which serves as an important factor in solution of incorrect tasks, and with increase of the number of equation included in the system (1+), relative error of calculation with use of these methods lowers. The wavelet transformation method allows us to reduce the condition number (μ) of the matrix S at $m = 3$, $p = 4$ from $8.4 \cdot 10^2$ down to $0.3 \cdot 10^2$, and down to $0{,}1 \cdot 10^3$ from $1.4 \cdot 10^3$ at $m = 8$, $p = 2$ and to obtain the mean value with respect to three samples not exceeding 0.40 % at $m = 3$, $p = 4$ and 0.17 % at $m = 8$, $p = 2$, which is obviously the best result.

3. DISCUSSION AND CONCLUSIONS

The article considers the methods of shooting of objects by means of an imaging spectrometer based on a multi-channel RD with corresponding software processing of the shooting results allowing to define SR of objects with increased spectral resolution and original spatial resolution by obtaining additional information on spectral characteristics of an object in the course of its shooting through special optical filters. Mathematical modelling of the shooting process confirming theoretical grounds of the method is conducted. In the course of the work it is found that the problem of results processing is not trivial, therefore the results of modelling of its solution using different methods are given in the article. It is found that the method of wavelet transformation is the most universal method of solution of the incorrect problem of object SR curves reconstruction. As a result of computational modelling, it is found that relative error of SR reconstruction based on the data of multi-channel shooting using the proposed method is about 0.17 % and is satisfactory. It is also shown that the discussed method of multi-channel imaging spectroscopy is applicable both to multispectral eight-channel matrix RDs and to widely used standard three-channel matrix RDs. When three-channel RDs are used, the number of optical filters used for shooting appears to be technically feasible and justified.

It appears that the main advantages of the described method of multi-channel shooting are simplicity of its technical implementation, low sensitivity to external factors (increased vibrations, significant temperature drops, etc.) and capability of the imaging spectrometer to obtain data in three coordinates of the cube without scanning with

Table. The Results of Modelling of Reconstruction of Spectral Radiance Based on the Data of Multichannel Shooting

Sample number	Relative error, %							
	Greville Method		Wavelet Transformation Method		Tikhonov Regularisation Method		Godunov Method	
	m=3	*m*=8	*m*=3	*m*=8	*m*=3	*m*=8	*m*=3	*m*=8
	p=4	*p*=2	*p*=4	*p*=2	*p*=4	*p*=2	*p*=4	*p*=2
1	0.96	2.86	0.62	0.25	1.33	0.77	0.65	1.61
2	0.91	2.55	0.29	0.11	1.20	0.61	0.42	1.33
3	0.61	2.00	0.30	0.15	1.11	0.56	0.55	1.54

high spectral and spatial resolution. The considered method allows a large number of variations (e.g. by changing the number of shooting channels and optical filters involved in it, transmission spectra of these filters, etc.). In consequence of such variability, optimisation of the values of the said parameters and their combinations as well as consideration of the effect of measurement errors on the results of object SR reconstruction using the data of indirect measurements should be the subject matter of further research.

REFERENCES

1. Golovin, A.D., Dyomin, A.V. The Imitation Model of the Multi-channel Offner Hyperspectrometer [Imitatsionnaya model mnogokanalnogo giperspectrometra Offnera] // Kompyuternaya optika, 2015, Vol. 39, # 4, pp. 521–528. Doi 10.18287/0134–2452–2015–39–4–521–528.

2. Larar Allen M. et al. Multispectral, hyperspectral, and ultraspectral remote sensing technology, techniques, and applications III // Proc. SPIE International Society for Optical Engineering. 13–14 October 2010. Incheon, Korea Republic, 2010, Vol. 7857.

3. Rodionov, I.D., Rodionov, A.I., Vedeshin, L.A., Vinogradov, A.N., Egorov, V.V., Kalinin, A.P. Aviation Hyperspectral Installations for Remote Sensing Applications [Aviatsionnyie giperspektralnyie kompleksy dlya resheniya zadach distantsionnogo zondirovaniya] // Issledovaniye zemli iz kosmosa, 2013, Vol. 6, pp. 81–93.

4. Gorbunov, G.G., Chikov, K.N., Shlishevskiy, V.B. Interferential Hyper and Ultraspectral Videospectrometers for Remote Sensing Applications [Interferentsionnyie giper- i ultraspektralnyie videospektrometry dlya zadach distantsionnogo zondirovaniya] // Bulleting of SGUGiT, 2016, Vol. 1, # 33, pp. 70–94.

5. Vilaseca M., Schael B., Delpueyo X., Chorro E., Perales E., Hirvonen T., Pujol J. Repeatability, reproducibility, and accuracy of a novel pushbroom hyperspectral system // Color Research & Application, 2013, Vol. 39, No. 6, pp. 549–558. Doi 10.1002/col.21851.

6. Pozhar, V.E., Machikhin, A.S., Gaponov, M.I., Shirokov, S.V., Mazur, M.M., Sheryshev, A.E. Hyper-spectrometer Based on an Acousto-optic Tuneable Filters for UAVS // Light & Engineering, 2019, Vol. 27, # 3, pp. 99–104.

7. Palto, S.P., Alpatova, A.V., Geyvandov, A.R., Blinov, L.M., Lazarev, V.V., Yudin, S.G. Fourier Spectroscopy as a Method of Studying of Photoelectric Properties of Organic Systems [Furye-spektroskopiya kak metod izucheniya fotoelektricheskikh svoistv organicheskikh sistem] // Optika i spektroskopiya, 2018, Vol. 124, # 2, pp. 210–220. Doi 10.21883/OS.2018.02.45526.209–17.

8. Zavarzin, V.I., Mitrofanova, Yu.S. Schematic Solutions for Advanced Hyperspectral Systems [Skhemnyie resheniya dlya perspektivnoy giperspektralnoy apparatury] // Opticheskiy zhurnal, 2017, Vol. 84, # 4, pp. 12–16.

9. Golovin, A.D., Dyomin, A.V. The Imitation Model of the Multi-channel Offner Hyperspectrometer [Imitatsionnaya model mnogokanalnogo giperspectrometra Offnera] // Kompyuternaya optika, 2015, Vol. 39, # 4, pp. 521–527. Doi 10.18287/0134–2452–2015–39–4–521–528.

10. Balashov, A.A., Vagin, V.A., Golyak, I.S., Morozov, A.N., Nesteruk, I.N., Khorokhorin, A.I. Visible and Near-IR Range Fourier Spectrometer [Furye-spektrometr vidimogo i blizhnego IK diapazonov] // Radiostroeniye, 2017, Vol. 6, pp. 27–38. Doi 10.24108/rdeng.0617.0000124.

11. Arkhipov, S.A., Zavarzin, V.I., Lee, A.V. // Reflective Optical Systems for Low-dimension Hyperspectral Systems of Remote Sensing of Earth [Zerkalnyie opticheskiye sistemy dlya malogabaritnoy giperspektralnoy apparatury distantsionnogo zondirovaniya zemli iz kosmosa] // From the collection Acoustic-optic and Radar Methods of Measurement and Information Processing. Proceedings of the 10th International Science and Tech-

nical Conference. A.S. Popov Russian Society of Radio Engineering, Electronics and Communication (NTORES), 2017, pp. 262–264.

12. Veys C., Davies P., Hibbert J., Grieve B. (2017). An Ultra-Low-Cost Active Multispectral Crop Diagnostics Device. 1005–1007. Paper presented at IEEE Sensors 2017 Conference, Glasgow, United Kingdom. Doi.org/10.1109/ICSENS.2017.8234211.

13. Burns P.D., Berns R.S. Analysis Multispectral Image Capture // Color Imaging Conferenct 1996, # 1, pp. 19–22.

14. Khorokhorov, A.M., Vvedenskaya, A.V., Shirankov, A.F., Kobozev, V.S., Algorithm of Enhancement of Matrix Radiation Detectors with Bayer filters [Algoritm rasshireniya vozmozhnostey matrichnykh priyomnikov izlucheniya s filtrami Bayera / Int. Conf. Applied Optics 2018, Saint Petersburg: D.S. Rozhdestvensky Society of Optics, 2018, Vol. 2.

15. Varentsova, S.A., Trofimova, V.A., Troshchiyev, Yu.V. Reconstruction of a Signal and Dynamics of its Spectral Characteristics with a Non-regular Set of Measurements [Vosstanovleniye signala i dinamiki ego spektralnykh kharakteristik pri neregulyarnom nabore izmereniy] // J. tech. phys. 2008, Vol. 78, # 7, pp. 57–68.

16. Zhuchko, O.V., Pytyev, Yu.P. Reconstruction of Functional Dependence Using Theoretical and Possibility Methods // J. comp. math. and math. phys. 2003, Vol. 43, # 5, pp. 767–783.

17. Hardeberg J. Acquisition and reproduction of color images: colorimetric and multispectral approaches. USA: Dissertation.com, 2001.

18. Vapnik, V.N. Reconstruction of Dependences Using Empirical Data [Vosstanovleniye zavisimostey po empiricheskim dannym], Moscow: Nauka, 1979, 448 p.

19. Ivanov, V.K., Vasin, V.V., Tanana, V.P. The Theory of Linear Incorrect Problems and its Application [Teoriya lineynykh nekorrektnykh zadach i eyo prilozhenie], Moscow: Nauka, 1978, 206 p.

20. Sizikov, V.S., Lavrov, A.V. Contemporary Sustainable Mathematical and Software Methods of Distorted Spectra Reconstruction [Sovremennyie ustoichivyie matematicheskiye i programmnyie metody vosstanovleniya iskazhonnykh spektrov] // Science and Technical Bulletin of Information Technologies, Mechanics and Optics, 2018, Vol. 18, # 6.

21. Sydikhov, A. Sh., Arapov, S. Yu., Arapova, S.P. Pseudoinverse Processing of the Data of Multispectral Shooting in Stationary Zones of an Image [Psevdoinversnaya obrabotka dannykh multispektralnoy fotosyomki v statsionarnykh zonakh izobrazheniya] / Int. Conf. of Students, Postgraduates and Young Scientists Information Technologies, Telecommunications and Control Systems: book of reports, Ekaterinburg: UrFU, 2015, pp. 179–185.

22. El-Rifai I. et al. Enhanced Spectral Reflectance Reconstruction Using Pseudo-Inverse Estimation Method // Int. J. Image Process. IJIP, 2013, Vol. 7, # 3, pp. 278–285.

23. Barliani, A.G. Development of Algorithms of Equalisation and Evaluation of Accuracy of Unconstrained and Constrained Geodetic Networks Based on a Pseudonormal Solution [Razrabotka algoritmov uravnivaniya i otsenki tochnosti svobodnykh i nesvobodnykh geodezicheskikh setey na osnove psevdonormalnogo resheniya, Novosibirsk: SGGA, 2010, 135 p.

24. Morozov, V.V., Grebennikov, A.I. Methods of Solution of Incorrectly Set Problems. The Algorithm Aspect [Metody resheniya nekorrektno postavlennykh zadach. Algoritmicheskiy aspekt]. – Moscow: Moscow University Press, 1992, 319 p.

25. Vogel C.R. Computational Methods for Inverse Problems. SIAM. Philadelphia, 2002, 179 p. Doi 10.1137/1.9780898717570.

26. Gurylyova, A.V., Khorokhorov, A.M., Latyshev V.I. Comparative Analysis of Methods of Solving Incorrect Reverse Problems for Multi-channel Hyperspectroscopy [Sravnitelnyi analiz metodov resheniya nekorrektnykh obratnykh zadach dlya mnogokanalnoy giperspektrometrii] // Optika i spektroskopiya, 2019, Vol. 127, # 4, pp. 551–557. Doi 10.21883/OS.2019.10.48356.171–19.

27. Arapov, S. Yu., Arapova, S.P., Dubinin, I.S., Sergeev, A.P. Reconstruction of Reflection Spectra of Test Fields Based on Data of Multispectral Shooting [Vosstanovleniye spektrov otrazheniya testovykh poley po dannym multispektralnoy fotosyomki] / Transmission, Processing, Perception of Text and Graphic Information: proceedings of the International Science and Practical Conference. – Ekaterinburg: UrFU, 2015. – P. 21–33.

28. Arapov, S. Yu., Tarasov, D.A., Sergeev, A.P., Kolmogorov, Yu.N. Modelling of Reflection Spectra Using the Basis of Erf Functions [Modelirovaniye spektrov otrazhaniya na osnove bazisa iz funktsiy tipa integrala oshibok] // Izvestiya vysshykh uchebnykh zavedeniy. Problemy poligrafii i izdatelskogo dela, 2012, Vol. 6, pp. 17–29.

29. Novikov, L.V. Spectral Analysis of Signals in Wavelet Basis [Spektralnyi analiz signalov v bazise veivletov] // Nauchnoye priborostroeniye. 2000, Vol. 10, # 3, pp. 70–77.

30. Tarasov, D.A. Modelling of Reflection Spectra by Polynome Superposition [Modelirovaniye spektrov otrazheniya superpozitsiey polinomov] // Izvestiya

vysshykh uchebnykh zavedeniy. Problemy poligrafii i izdatelskogo dela, 2012, Vol. 5, pp. 59–66.

31. Tikhonov, A.N., Arsenin, V. Ya. Methods of Solving of Incorrect Problems [Metody resheniya nekorrektnykh zadach]. Moscow: Nauka, 1986, 288 p.

32. Calvetti, D., Morigi, S., Reichel, L., Sgallari, F. Tikhonov regularization and the L-curve for large discrete ill-posed problems // J. Comput. Appl. Math. 2000, Vol. 123, pp. 423–446.

33. Hansen, C.A. Matlab Package for Analysis and Solution of Discrete Ill-Posed Problems // Numerical Algorithms, 1994, Vol. 6, pp. 1–35.

34. Morozov, V.A. Linear and Non-linear Incorrect Problems [Lineynyie i nelineynyie nekorrektnyie zadachi] // Itogi nauki i tekhniki. Ser. Math. anal. 1973, Vol. 11, pp. 129–178.

35. Goncharskiy, A.V., Leonov, A.V., Yagola, A.G. Generalised Residual Principle [Printsip obobshchyonnoy nevyazki] // J. Comp. Math. & Math. Phys. 1973, Vol. 13, # 2, pp. 294–302.

36. Godunov, S.K., Antonov, A.G., Kirilyuk, O.P., Kostin, V.I. Guaranteed Accuracy of Solution of a System of Linear Equations in Euclidean Spaces [Garantirovannaya tochnost resheniya sistem lineynykh uravneniy v evklidovykh prostranstvakh]. Novosibirsk: Nauka: Sib. branch, 1988, 456 p.

37. Vapnik, V.N., Mikhalskiy, A.I. On Searching of Dependences Using the Method of Ordered Risk Minimisation [O poiske zavisimostey metodom uporyadochennoy minimizatsii riska] // Automat. & telemech. 1974, Vol. 10, pp. 10–21.

38. Connah, D., Alsam, A., Hardeberg, J.Y. Multispectral imaging: How many sensors do we need? // J. Imaging Sci. Technol. 2006, Vol. 50, #. 1, pp. 45–52. Doi 10.2352/j.imagingsci.technol. (2006)50:1(45).

39. Helling, S., Seidel, E., Biehlig, W. Algorithms for spectral color stimulus reconstruction with a seven-channel multispectral camera / Proc. of CGIV (2nd European Conference on Color in Graphics, Imaging and Vision). 2004, Vol. 2, pp. 254–258.

40. Imai, F.H., Berns, R.S. Spectral estimation using trichromatic digital cameras // Proc. Int. Symp. Multispectral Imaging and Color Reproduction for Digital Archives. Chiba: Society of Multispectral Imaging of Japan, 1999, pp. 42–48.

41. Masahiro, Y., Hideaki, H., Nagaaki, O. Beyond Red–Green–Blue (RGB): Spectrum-Based Colour Imaging Technology // J. Imaging Sci. Technol. 2008, Vol. 52, #. 1, pp. 1–15.

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