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RESEARCH INTO METHODS FOR DETERMINING COLOUR DIFFERENCES IN THE *CIELAB* UNIFORM COLOUR SPACE

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ABSTRACT

A problem of qualitative determination of the results of colour image capturing by digital devices based on uniform colour systems arises during analysis of colour images. The article describes methods of determining colour difference as recommended by CIE for the *CIELAB* system, presents mathematical formulation of the system and its modifications, describes the programme developed for calculating colour difference for each described method, and describes testing of this programme using 14 sample colours from the Munsell Book of Colour. Three groups of red, green and blue filters with 7 filters in each were selected as the study object. The filters were selected based on possibility of their further application for determining colour shift by digital devices. Filter colour difference was studied in the groups with different standard sources of light and using three methods of calculation of this difference. Analysis of the obtained results is presented and conclusions are made on possibility to apply each method in different industrial spheres. The article may be useful for colour analysis specialists, quality inspectors and specialists in digital devices and computer vision.

Keywords: colour, colour difference, uniform colour system, colorimetric system, filters, quality control, estimation, computer vision

1. INTRODUCTION

Determination of colour difference is still necessary in light engineering, chemistry, non-resource industry, printing industry, diamond colorimetric and other fields of science and engineering. CIE defined several uniform colorimetric systems and several methods of determining colour shift. The formulae for calculating colour difference allow us to use colorimetry to solve the following problems: sorting by colour, unbiased evaluation of colour stability, defining unbiased colour tolerances and checking them, high-quality printing. There is no common universal solution of the problem of colour difference evaluation. Modern colour determination systems used in engineering and industry are based on colour matching experiments. Each consumer uses any proposed method which is convenient and clear in any particular case. Production technologies are constantly developing, computer vision is being matured, but there is still no uniform method of colour registration and reproduction quality evaluation.

Such uncertainty causes a number of problems of adequate evaluation of object colour in different spheres of industry. Measurement of colour difference allows us to estimate unbiased the reproduction accuracy and to predict possible mistakes. A necessity of introducing colour quality management systems and instrumental methods of colour control arises. Most of the problems are faced by users of modern digital devices during comparison of screen colour with colour reproduced by actual reflecting materials such as paint, a print publication, or textile [1, 2].

Nowadays, the *CIELAB* (also designated as $L^*a^*b^*$) uniform colour system has been used with increased frequency for unbiased colour evalua-

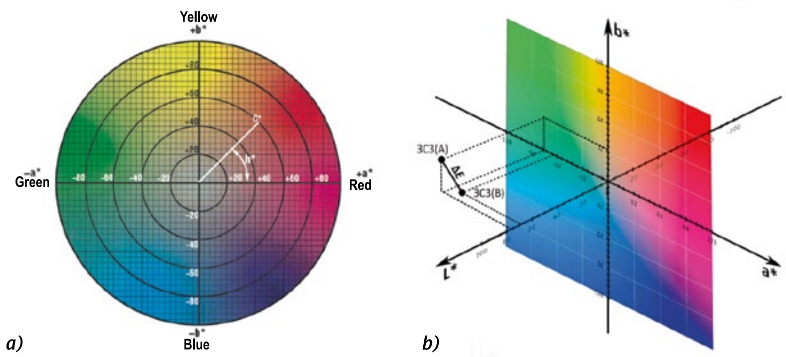


Fig. 1. *CIELAB* with polar coordinates *C* and *H* (*a*) and colour difference of the 3C3 filter in it (presented as the Euclidean distance) with standard *A*-type and *B*-type sources of light

tion. This system is used more frequently than, for instance, the *CIELUV* system (also designated as $L^*u^*v^*$) despite a number of advantages of the latter [3]. The relatively new system $CIECAM_{02}$ is still not spread as widely as the above mentioned ones due to its complex mathematical formulation. Although same-tone curves in blue-violet and magenta colours are completely corrected and more stable hue angle straightening through different levels of luminance as per Munsell colours is observed in general, as compared to *CIELAB*, $CIECAM_{02}$ requires compliance with additional conditions during colour evaluation, such as knowledge of average luminance of visual surroundings and adaptive luminance of the background surrounding a colour object. All this causes difficulties of comprehension and, therefore, application of the system by users.

According to decisions made by CIE, different colour difference ΔE calculation formulae were proposed for the *CIELAB* system (expressed in the form of Euclidean distance between points). Three methods of colour difference calculation were developed. Each further calculation became more difficult due to new coefficients. But is it really necessary to introduce new magnitudes thus complicating the expressions? Which method will be more efficient and convenient for colorimetric use, for instance, just as a way to analyse quality of products?

It was necessary to study the three methods of colour shift determination in the *CIELAB* uniform colour system and to define the most accurate and universal one. This study was based on mathematical modelling of major provisions of colorimetry, colorimetric systems and their transformation.

2. ANALYSIS OF *CIELAB* SYSTEM CALCULATION METHODS

Unbiased evaluation of colour difference is based on possibility to represent colours by means of particular points which are endpoints of vectors situated close to each other in a colour space. With increase in the Euclidean distance between the points, the colour difference will increase too. This means that it would be safe to assume ΔE for unbiased evaluation of the level of colour difference between natural objects. Example of change in chromaticity of an object when illuminated by different sources of light (of types *A* and *B* in the example) is shown in Fig. 1, *a*.

Let us consider different formulae for calculation of ΔE in the *CIELAB* system.

In 1976, CIE recommended the formula

$$\Delta E_{1976} = \left[(\Delta a)^2 + (\Delta b)^2 + (\Delta L)^2 \right]^{1/2},$$

where $\Delta a = a_1^* - a_2^*$, $\Delta b = b_1^* - b_2^*$, $\Delta L = L_1^* - L_2^*$.

It allowed colour differences to determine relatively easily but it had its own flaws. The *CIELAB* system is a curvilinear transformation of the CIE *XYZ* system, therefore such approach to determination of colour shift is fundamentally flawed.

With consideration of it, CIE proposed a modification of the *CIELAB* system itself, *CIELCH*. Polar coordinates of hue *H* and chromaticity *C* (Fig. 1) were proposed. The coordinate *H* is designated as $h°_{ab}$ and is the polar angle whereas the coordinate *C* is designated as $\tilde{N}^*_{ab}$ and is the radial coordinate in relation to the coordinate centre (L^* axis):

$$\tilde{N}^*_{ab} = \left(a^{*2} + b^{*2} \right)^{1/2},$$

$$h^*_{ab} = \text{arctg}(b^* / a^*).$$

Such modification allowed us to describe colour in this system as the process of colour presentation by human eye.

That is why, to increase accuracy, CIE approved the new formula in 1994

$$\Delta E_{1994}=\left[\left(\frac{\Delta L}{K_L S_L}\right)^2+\left(\frac{\Delta C}{K_C S_C}\right)^2+\left(\frac{\Delta H}{K_H S_H}\right)^2\right]^{1/2},$$

where $\Delta L = L_1^* - L_2^*$; $\Delta C = C_1^* - C_2^*$;

$\Delta H = \sqrt{\Delta a^2 + \Delta b^2 - \Delta C^2}$; $K_L = 1$, $K_C = 1$ and $K_H = 1$ (weighting factors are normally equal to one but may take particular values for graphics, printing industry, textile industry, etc.), $S_L = 1$, $S_C = 1 + K_1 C_{12}^*$, $S_H = 1 + K_2 C_{12}^*$, $L_{12}^* = \left(L_1^* + L_2^*\right)/2, C_{12}^* = \left(C_1^* + C_2^*\right)/2$.

Then the formula was modified and CIE approved the new one in 2000:

$$\Delta E_{2000}=\left[\begin{aligned}&\left(\frac{\Delta L}{K_L S_L}\right)^2+\left(\frac{\Delta C}{K_C S_C}\right)^2+\left(\frac{\Delta H}{K_H S_H}\right)^2+\\&+R_T\left(\frac{\Delta C}{K_C S_C}\right)\left(\frac{\Delta H}{K_H S_H}\right)\end{aligned}\right]^{1/2},$$

where $\Delta H = 2\sqrt{C_1^* C_2^*}\sin\left[\frac{H_1^* - H_2^*}{2}\right]$,

$$S_L = 1 + \frac{K_2\left(L_{12}^* - 50\right)^2}{\sqrt{20 + \left(L_{12}^* - 50\right)^2}},\ H_{12}^* = \left(H_1^* + H_2^*\right)/2,$$

$$S_H = 1 + K_2 C_{12}^* T,\ R_T = -\sin\left(2\Delta\theta\right)R_C,$$

$$T = 1 - 0{,}17\cos\left(H_{12}^* - 30°\right) + 0{,}24\cos\left(2H_{12}^*\right) + $$
$$+ 0{,}32\cos\left(3H_{12}^* + 6°\right) - 0{,}20\cos\left(4H_{12}^* - 64°\right),$$

$$\Delta\theta = 30\exp\left\{-\left[\left(H_{12}^* - 275°\right)/25\right]^2\right\},$$

$$R_C = 2\sqrt{\frac{C_{12}^{*\,7}}{C_{12}^{*\,7} + 25^7}}.$$

The formulae of 2000 include more complex parameters of saturation, hue and brightness but analysis of the obtained Euclidean distance is performed in the same way as in the previous method. Colour differences are considered in relation to determination of tolerances within which colour matching is observed. For instance, in the *CIELAB* colour space, built for tolerances aligning for the task of colour matching determination, colours are considered equal for visual perception if the distance ΔE between them does not exceed 1 unit.

3. PROGRAMME DEVELOPMENT AND TESTING

For calculation and analysis of colour differences in the *CIELAB* system, a programme was developed in the *MATLAB* environment to set formulae for determining colour shifts ΔE_{1976}, ΔE_{1994} and ΔE_{2000}.

The algorithm used for the programme is distinctive as it complies with a particular condition: if chromaticity coordinates of samples are in different quadrants, it is necessary to take into account that at $H_{12}^* \geq 180°$, 360° should be subtracted from the value of this coordinate and then the mean value should be determined during calculation. Correction of the a^* value of the coordinate is taken into account too:

$$\grave{a}^* = \left(1 + G\right)a_o,$$

$$G = 0{,}5\left(\sqrt{\frac{C_{o\ 12}^{*7}}{C_{o\ 12}^{*7} + 25^7}}\right),$$

$$C_o^* = \sqrt{a_o^{*2} + b_o^{*2}},$$

where a_o^* and b_o^* are the values determined in the course of transition from the CIE *XYZ* system to the *CIELAB* system with consideration of chromaticity coordinates of the source of white light. The latter fact demonstrates that the chromaticity coordinates of the system are no longer independent as they include the luminance factor of the source of light.

In the course of testing, coordinates of control samples of 14 colours from the Munsell Book of Colours for evaluation of colour rendering described in the well-known calculation methodology [4] were taken as input data. The following colours were taken as samples: grey-red, grey-yellow, yellow-green, light-green, light-blue, bright-blue, light-violet, magenta, red, yellow, green, blue, pink (colour of skin) and green (colour of leaves). A standard source of light was taken as reference radiation. Tristimulus values were recalculated for standard *A*-type and *B*-type sources as per Russian standard GOST R55703–2013 from the CIE *XYZ* system into the *CIELAB* system using the following formulae:

$$L^* = 116\left(Y/Y_o\right)^{1/3} - 16,$$

Table 1. Values of ΔE in the *CIELAB* Space for *A* and *B* Type Sources of Light

ΔE	Sample number														Mean ΔE
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
2000	12.53	18.87	51.67	33.43	42.05	85.07	28.33	17.20	2.95	18.69	18.80	11.86	14.58	54.42	29.3
1994	32.01	37.07	43.30	46.22	54.13	62.69	55.30	40.51	7.70	29.62	34.20	17.09	35.52	45.81	38.7
1976	99.99	98.06	99.11	110.74	106.86	116.43	124.59	122.41	45.57	87.26	113.72	57.53	99.90	99.93	98.7

Table 2. Values of ΔE in the *CIELAB* Space for *C* and D_{65} Type Sources of Light

ΔE	Sample number														Mean ΔE
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
2000	1.73	2.15	0.78	0.20	1.18	2.47	1.17	0.89	1.13	2.94	0.73	1.53	3.34	0.46	1.48
1994	2.23	2.04	1.38	0.22	1.63	2.46	2.09	1.87	1.43	3.16	1.24	1.16	3.52	0.87	1.81
1976	4.53	4.55	4.05	1.09	3.90	8.09	5.29	3.00	3.54	13.92	3.45	3.49	8.62	1.17	4.91

$$a^{*}=500\left[\left(X/X_{o}\right)^{1/3}-\left(Y/Y_{o}\right)^{1/3}\right],$$

$$b^{*}=200\left[\left(Y/Y_{o}\right)^{1/3}-\left(Z/Z_{o}\right)^{1/3}\right],$$

where X_o, Y_o, Z_o are tristimulus values of a corresponding source of light; X, Y, Z are tristimulus values of a sample.

Calculation of colour difference of tristimulus values with the selected sources of light presents an opportunity to analyse all the three considered CIE methods (Table 1). The mean value of ΔE_{1976} differs from the mean value of ΔE_{1994} by a factor of 2.6, the mean value of ΔE_{1976} differs from the mean value of ΔE_{2000} by a factor of 3.4, and the mean value of ΔE_{1994} differs from the mean value of ΔE_{2000} by a factor of 1.3. The Euclidean distance between points ΔE calculated using the 1976 formula is outstandingly higher that those calculated using the 1994 and 2000 formulae, which is as expected due to nonlinear nature of the chromaticity scale.

With illumination by means of different sources, the difference becomes noticeable and perceived even visually. The calculated values of ΔE exceed the colour matching tolerances by ten folds what means that additional calculations will be required.

4. STUDY OF COLOUR DIFFERENCE

For detailed analysis and evaluation of the obtained results, it was proposed to take samples of

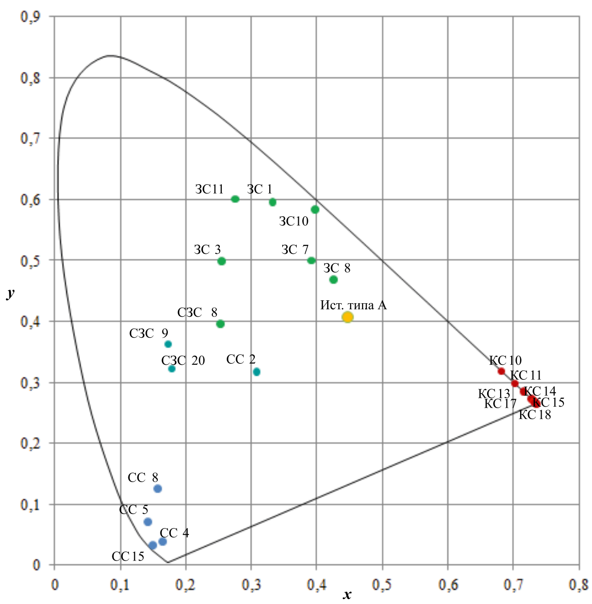
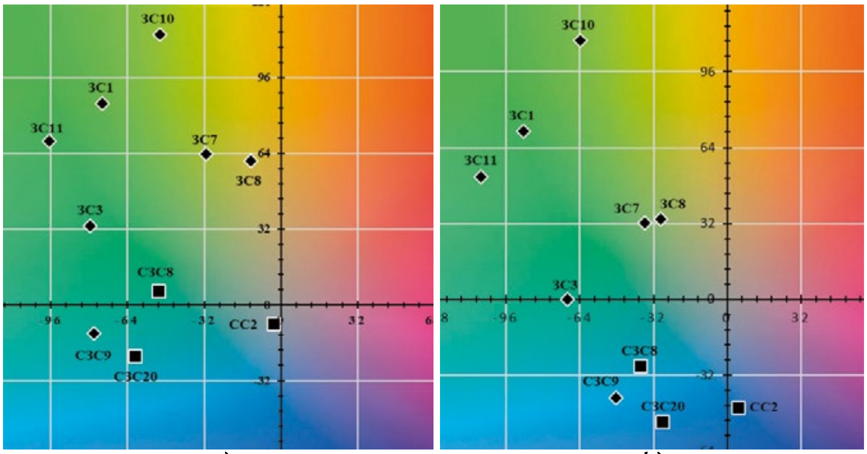


Fig. 2. Coordinates of the studied filters in the *x*, *y* colour space with *A*-type source



a) b)

Fig. 3. Coordinates of the studied green filters illuminated in the *CIELAB* colour space by *A*-type (*a*) and *B*-type (*b*) sources of light

three reference colour stimuli: red, green and blue. As the results of the work will be used for studying an actual *A*-type source of light and colorimetric systems, actual objects were selected from the set of coloured optical glasses compliant with Russian standard GOST 9411–91. The obtained results will be further taken as reference values for studying the selected filters by means of an optoelectronic assembly. Due to the limited filter set, the coordinates of 7 red filters (KC10, KC11, KC13, KC14, KC15, KC17, KC19), 7 green filters (3C1, 3C3, 3C7, 3C8, C3C8, 3C10, 3C11) and 7 blue filters (CC2, CC4, CC5, CC8, CC15, C3C9, C3C20) were taken as reference data for the study. These filters and the A source are presented in the *x*, *y* colour space (Fig. 2).

The chromaticity coordinates of these filters are provided in the catalogue[1] in relation to *A* and *B* type sources. So the latter ones were taken as main coordinates to prevent errors during recalculation of colour in relation to other sources of light into the study being conducted (Fig. 1, *b*). The chromaticity coordinates *x* and *y* should be transformed into tristimulus values *XYZ* using the expression

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \frac{Y}{y}x \\ Y \\ \frac{Y}{y}(1-x-y) \end{pmatrix}.$$

Changes of the coordinates of the selected filters with illumination by different sources of light are quite visible in the *CIELAB* colour space (Fig. 3).

Using the programme, colour differences were determined using each of the considered methods for the selected sets of red, green and blue filters with two standard sources of light: *A*-type and *B*-type (Tables 2–4). The calculated values of colour difference demonstrate that some samples systematically lower rank of colour differences as compared to the mean value while the others constantly increase it.

Δ*E* evaluations showed that, on average, the results of the calculations using the 1976 formula: 1) are higher by the factor of 2 than those obtained by the 1994 and 2000 formulae for the group of green filters; 2) are higher by the factors of 2.2 and 2.6 than those obtained by the 1994 and 2000 formulae respectively for the group of blue filters; 3) are on average higher by the factor of 3.2[2] than those obtained by the 1994 and 2000 formulae for the group of red filters. Moreover, the tables generally demonstrate that the difference between calculation methods using the 1994 and 2000 formulae is insignificant.

CONCLUSION

The obtained calculation results show that the CIE method of 1976 should not be used for the *CIELAB* system as it does not take its distinctions into account. It is appropriate for simple $L^*u^*v^*$ and $W^*u^*v^*$ systems which have more linear na-

[1] Coloured Optical Glass and Special Glasses. Catalogue [Tsvetnoye opticheskoye steklo i osobyie styokla. Katalog] / Edited by prof. G.T. Petrovskiy. Moscow: Dom optiki, 1990. 228 p.

[2] Such high difference may be explained by distinctions of the *CIELAB* system itself which is relatively “stretched” in the red shades region and compressed in the blue-green region [3].

Table 3. Colour Difference of Red Filters

Type of filter	Colour shifts			ΔE comparison		
	ΔE_{1976}	ΔE_{1994}	ΔE_{2000}	$\Delta E_{1976}/\Delta E_{1994}$	$\Delta E_{1976}/\Delta E_{2000}$	$\Delta E_{1994}/\Delta E_{2000}$
KC10	2.08	0.63	0.79	3.3	2.6	0.8
KC11	0.97	0.25	0.35	3.9	2.8	0.7
KC13	9.44	2.82	2.60	3.3	3.6	1.1
KC14	0.19	0.06	0.06	3.3	3.0	0.9
KC15	0.06	0.04	0.02	1.5	3.0	2.0
KC17	0.13	0.05	0.04	2.3	3.0	1.3
KC18	0.32	0.15	0.11	2.2	3.0	1.4
Mean value	1.9	0.6	0.6	3.3	3.3	1.0

Table 4. Colour Difference of Green Filters

Type of filter	Colour shifts			ΔE comparison		
	ΔE_{1976}	ΔE_{1994}	ΔE_{2000}	$\Delta E_{1976}/\Delta E_{1994}$	$\Delta E_{1976}/\Delta E_{2000}$	$\Delta E_{1994}/\Delta E_{2000}$
3C11	20.15	7.36	6.29	2.7	3.2	1.2
3C7	31.93	18.02	15.97	1.8	2.0	1.1
3C10	14.11	5.03	7.94	2.8	1.8	0.6
C3C8	42.68	22.40	19.80	1.9	2.2	1.1
3C8	31.34	18.08	20.55	1.7	1.5	0.9
3C3	35.53	17.82	13.07	2.0	2.7	1.4
3C1	20.09	7.46	8.05	2.7	2.5	0.9
Mean value	28	13.7	13.1	2.0	2.1	1.0

Table 5. Colour Difference of Blue Filters

Type of filter	Colour shifts			ΔE comparison		
	ΔE_{1976}	ΔE_{1994}	ΔE_{2000}	$\Delta E_{1976}/\Delta E_{1994}$	$\Delta E_{1976}/\Delta E_{2000}$	$\Delta E_{1994}/\Delta E_{2000}$
CC2	39.98	14.02	13.71	2.9	2.9	1.0
CC4	19.16	7.40	4.16	2.6	4.6	1.8
C3C9	37.78	22.87	21.89	1.7	1.7	1.0
C3C20	45.77	25.32	21.21	1.8	2.2	1.2
CC8	40.06	19.18	15.20	2.1	2.6	1.3
CC5	27.40	11.31	8.77	2.4	3.1	1.3
CC15	11.356	4.05	2.51	2.8	4.5	1.6
Mean value	31.6	14.9	12.5	2.1	2.5	1.2

ture in relation to *XYZ*. This calculation method slightly overestimates colour shifts, which may lead to wrong interpretation of object colour changes.

The 2000 CIE method is the most accurate one for colour shift evaluation as it takes more nuances of colour description into account. However, it is necessary in cases when the goal is to describe the colour itself: in printing, colorimetry, graphics, etc., while the main industrial problem solved by means of colorimetric systems is quality control. Determination of colour difference is one of the methods of compliance with Russian standards (GOST and

SanPiN), so the CIE1994 method is more appropriate for product quality control as its results are similar to those of the CIE2000 method but it also includes all variables necessary for accurate calculation and is not complicated by additional parameters and calculations.

It is also necessary to mention low-saturated and low-comprehensible colours. The CIE1976 method is more frequently used for such objects evaluation. However, the angle and radius also change in relation to the centre of coordinates in these colours and it is necessary to take it into account too. So, the CIE1994 method is more optimal for such colours.

In the future, it is possible to compare *CIELAB* and *CIECAM* uniform colour systems though *CIELAB* has already proved itself as the most clear and convenient system, which is especially important for users of colorimetric measurements as a method of quality control and development of products. A corresponding test of analysis of colour differences between objects used for colour processing and changing may be used for unbiased evaluation of quality of colour registration and reproduction by modern digital devices [5].

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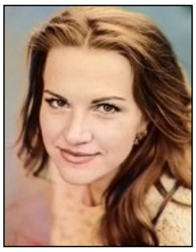

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