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THE NATURE OF THE PHOTON AND QUANTUM OPTICS

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Everything has to be made so simple, as possible, but not easier
A. Einstein

ABSTRACT

In a concise, but accessible for the first acquaintance form the procedure for the quantization of linear oscillator is set out. By analogy with this procedure the procedure of quantization (second quantization) of classical Maxwell's electrodynamics is set up. The physical sense of the wave functions arguments of transverse electromagnetic field and its Fourier transformation is set up. One pay attention as for quantum coherent (almost a classic) states of the electromagnetic field and for photonics Fock states. Attention is drawn to the fact of absence the power of the universal content of such concepts as field amplitude, phase and number of particles (photons), which are used by experimenter's to describe the states of a quantized field. The semi quantitative description the interaction processes of a quantum electromagnetic field with substance is set up. Specified situations are shown in which the discrepancy between the predictions of classical and quantum electrodynamics is noticeable at the macroscopic level.

Keywords: classical electrodynamics, quantum electrodynamics, quantum coherent states, Fock states, photon

1. INTRODUCTION

About a hundred years ago, a famous French physicist L. de Broglie (1892–1987) posed the problem to describe the diffraction and interference photon properties, which existence was theoretically predicted by M. Planck (1858–1947) in 1905. On the basis of his theory [1] de Broglie put the then-known equations $E = \hbar\omega$ and $E = mc^2$. This way led him to the conclusion that the photon possesses a small, but a finite mass. At the same time it remains unclear what are the principal differences between the photon and the other "massive" elementary particles. And then de Broglie came up to the brilliant idea: if the fundamental differences between the particles are absent than, on the contrary, all "massive particles" like a photon have to possess the wave properties. In such way the quantum mechanics of particles of the final mass was constructed. But formulated above the primary problem hasn't been fulfilled by de Broglie. The theory of a photon possessing both corpuscular and wave properties, was constructed later, in five years, by the work of other scientists. This theory turned out to be rather rich and complex, requiring the common efforts of many people. Below in elementary terms one presents its main concepts.

The idea of the light and its internal structure by the development of scientific knowledge has undergone dramatic perturbations. The concept of "ray of light" has ancient origin. There are two author of the law of refraction: V. Snellius (1580–1626) and R. Descartes (1596–1650). In works of C. Huygens (1629–1695), R. Hooke (1635–1703), and particle I. Newton (1643–1727) was opened the wave nature of light. But the date of establishment of this theory should considered the 1865, when J.C. Maxwell (1831–1879) got from his theory of electricity and magnetism the conclusion of the existence of electromagnetic waves. Describing by electric E^{ν} (r,t) and magnetic $H^{\nu}(r,t)$ strength posed at each point of space r at any time t, these waves perfectly definite quantity the interference, diffraction and the polarization of light. As for radiation and absorption of light, this theory is coming across the difficulties.

Investigating this question, firstly from thermodynamic point of view, and then developing the hypothesis of L. Bolsman (1844–1906) about the inevitability in the nature of the jump-figurative processes, M. Planck introduced the concept "photon" into the theory, which was reported by him at a meeting in German Physical Society 14 December 1905 year. Introduction the "photon" has made possible the understanding of atomic phenomena. As for the properties of the photon until the end of the twentieth years of last century one know nothing apart the expressions for its energy $\hbar\omega$ (ω is the frequency) and momentum $\hbar\mathbf{k}$ ($\mathbf{k}$ is the wave vector). As soon as one speaks about the photons, he/she had to forget about the attributes of Maxwell theory $E^{\nu}(r,t)$ and $H^{\nu}(r,t)$. So the Planck's constant $\hbar = 1{,}05\cdot 10^{-27}\,'erg\cdot s$ came to the theory. But rather soon it became clear that this constant is applicable not to light only, but also to bodies of finite mass.

In the works in the first place N. Bohr (1885–1962), L. de Broglie, W. Heisenberg (1901–1976) and E. Schrödinger (1887–1961) the quantum mechanics of particles of final mass was constructed. It was this theory that pointed the way and the need of building a consistent quantum theory of optical phenomena. The necessity of such theory directly follows from the definition of electric strength $E^{\nu}(r,t)$ as the force acting from the force on a single point charge. In quantum theory, any charged body describes by the wave function, the concept of localization blurs. Together with it, the classical concept of electric strength is blurred. The quantum theory of the electromagnetic field was built in the papers of P. Dirac (1902–1984), V. Heisenberg, V. Pauli (1900–1958), P. Jordan (1902–1980) and E. Fermi (1901–1954) in the period from 1927 to 1930. This theory combines classical (wave) and quantum (corpuscular) properties of photons. On a question "What is a photon?" this theory gives a clear answer: "Photon is an electromagnetic object described by its wave function". The explicit form of the wave function is well known, whereas with her interpretation the situation is more complicated. Here are stored all problems, characteristic of quantum mechanics wave function of particles of finite mass, often discussed up to now [2]. But at the same time be more questions about the methods of its calculation. In spite of the fact that such a theory is now well developed, in engineering practice it still has not found its application. This review aims to introduce the reader to enter into the non-trivial circle of quantum theory of electromagnetic field ideas. I want to believe that, if the reader with pencils in hand will review the proposed review of more than one under consideration, it will significantly facilitate him /her further acquaintance with voluminous manuals [3] on the subject.

2. THE CLASSICAL ELECTROMAGNETIC FIELD AND THE QUANTUM OSCILLATOR

According to Maxwell's equations, electromagnetic waves characterizing the propagation of light in the vacuum can be described by introducing the vector potential $A^{\nu}(r,t)$ that satisfy the wave equations [4]:

$$\nabla^2 A^{\nu}(r,t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}A^{\nu}(r,t) = 0. \quad (1)$$

Instead of the vector potential one traditionally use vectors

$$E^{\nu}(r,t) = -\frac{1}{c}\frac{\partial}{\partial t}A^{\nu}(r,t) \text{ and}$$
$$H^{\nu}(r,t) = rotA^{\nu}(r,t). \quad (2)$$

The solution of equation (1) can be represented in the form of superposition of plane waves

$$A^{\nu}(r,t) = \sum_{\mathbf{k}\lambda} e^{\nu}_{\mathbf{k}\lambda}\gamma_k\left(\alpha_{\mathbf{k}\lambda}e^{i\mathbf{kr}-ikct} + \alpha^{*}_{\mathbf{k}\lambda}e^{-i\mathbf{kr}+ikct}\right), \quad (3)$$

where $\mathbf{k}$ is the wave vector with components $\{k_x, k_y, k_z\}$, $\alpha_{\mathbf{k}\lambda}$ is dimensionless constants that depends on wavelength $\lambdabar = 2\pi/k$ and the index of transverse $(ke_{k\lambda}) = 0$ polarization $\lambda = 1,2$. By $e^{\nu}_{\mathbf{k}\lambda}$ the unit vectors are designated perpendicular to the direction of wave propagation that is to vector $\mathbf{k}$. Next, c is the speed of light in vacuum, and the functions γ_k, having the dimension of a vector potential are introduced for the convenience of further consideration. If one put $\alpha_{\mathbf{k}\lambda} = |\alpha_{\mathbf{k}\lambda}|\exp(i\vartheta_{\mathbf{k}\lambda})$ we get:

$$A^{\nu}(\mathrm{r},t)=\sum_{\mathbf{k}\lambda}e^{\nu}_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}\cos\left(\mathbf{kr}-kct+\vartheta_{\mathbf{k}\lambda}\right),$$
$$A_{\mathbf{k}\lambda}=2\left|\alpha_{\mathbf{k}\lambda}\right|\gamma_{k}. \quad (4)$$

According to (4) the classical electromagnetic waves described by entering the classical oscillator at each point of space $\mathbf{r}$, which performs the harmonic oscillations $A_{\mathbf{k}\lambda}\cos\left(\mathbf{kr}-kct+\vartheta_{\mathbf{k}\lambda}\right)$ with the frequency $\omega_k = kc$. This suggests that one describes the quantum the electromagnetic wave then the classical oscillators must be replaced by quantum one.

Recall that under the classical oscillator one means the material point with mass m, which coordinate $x(t)$ under the action of elastic forces $f(t)=-\chi x(t)$ performs the harmonic oscillations with the frequency ω. From the Newton's law $m\left(d^2/dt^2\right)x(t)=-\chi x(t)$ it follows that

$x(t)=x_0\cos\left(\omega t+\vartheta\right)$ where $\omega=\sqrt{\chi/m}$.

How to construct a quantum theory of the oscillator? In quantum theory is to replace the coordinate of the material x and its momentum p are given by operators $x\rightarrow\hat{x}\equiv x$ and $p\rightarrow\hat{p}=-i\hbar\partial/\partial x$. By such replacement the expression of classical point total energy E has to be replaced by operator expression named "Hamiltonian" or the total energy operator $\hat{H}$

$$E=\frac{p^2}{2m}+\frac{\chi}{2}x^2=$$
$$=\frac{p^2}{2m}+\frac{m\omega^2}{2}x^2\rightarrow-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{m\omega^2}{2}x^2=\hat{H}.$$

In quantum mechanics the state of a quantum particle is described by the wave function $\psi(x)$ obeying the equation to the name of E. Schrödinger:

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{m\omega^2}{2}x^2\right)\psi(x)=E\psi(x) \text{ or}$$
$$\hat{H}\psi(x)=E\psi(x). \quad (5)$$

The square of the wave function $\psi^*(x)\psi(x)dx$ determines the probability of detection of the classical coordinates of the particle in the interval dx around the point x, if such experiment will be delivered. Therefore,

$$\int_{-\infty}^{\infty}\psi^*(x)\psi(x)\,dx=1.$$

We write several formal properties of the quantum oscillator [5]. After the change the variables

$$\varsigma=x\beta,\ \beta=\sqrt{\frac{\omega m}{\hbar}},\ \lambda=\frac{2E}{\hbar\omega},$$

the Schrödinger equation (5) takes the form

$$\left(\varsigma^2-\frac{d^2}{d\varsigma^2}\right)\psi(\varsigma)=\lambda\psi(\varsigma).$$

Of course, instead of $\psi(\varsigma)$ could be used $\psi\left(x\sqrt{m\omega/\hbar}\right)$, but it is inconvenient. The expression in brackets resembles an algebraic difference of squares. That is why is seems to be the natural to use the operators

$$\hat{\alpha}=\frac{1}{\sqrt{2}}\left(\varsigma+\frac{\partial}{\partial\varsigma}\right),\ \hat{\alpha}^+=\frac{1}{\sqrt{2}}\left(\varsigma-\frac{\partial}{\partial\varsigma}\right).$$

By means of these operators the Schrödinger equation may be rewritten the form

$$\frac{\hbar\omega}{2}\left(\hat{\alpha}^+\hat{\alpha}+\hat{\alpha}\hat{\alpha}^+\right)\psi(\varsigma)=E\psi(\varsigma),$$
$$\hat{H}=\frac{\hbar\omega}{2}\left(\hat{\alpha}^+\hat{\alpha}+\hat{\alpha}\hat{\alpha}^+\right). \quad (6)$$

Under additional condition

$$\int_{-\infty}^{\infty}\psi^*(\varsigma)\psi(\varsigma)d\varsigma=1$$

this equation has many solutions, determined by the index $n=0,1,2,...$ Let us denote these solutions through $\varphi\left(\varsigma|n\right)$

$$\varphi(\varsigma|n)=H_n(\varsigma)e^{-\frac{\varsigma^2}{2}},$$
$$H_n(\varsigma)=\frac{(-1)^n}{\sqrt{2^n n!\sqrt{\pi}}}e^{\varsigma^2}\frac{d^n}{d\varsigma^n}e^{-\varsigma^2}. \quad (7)$$

Function $H_n(\varsigma)$ is called the Hermit polynomials. To each solution $\varphi(\varsigma|n)$ its own parameter E corresponds, determing permissible discrete values of the oscillator energy $E_n=\hbar\omega\left(n+1/2\right)$, if on it there are no external forces. Functions (7) are found to be real and possessing the following properties

$$\hat{\alpha}\varphi(\varsigma|n)=\sqrt{n}\varphi(\varsigma|n-1),$$
$$\hat{\alpha}^{+}\varphi(\varsigma|n)=\sqrt{n+1}\varphi(\varsigma|n+1), \qquad (8)$$
$$\int_{-\infty}^{\infty}\varphi(\varsigma|n)\varphi(\varsigma|n')d\varsigma=\delta_{nn'}.$$

It follows from these equations that

$$\hat{\alpha}^{+}\hat{\alpha}\varphi(\varsigma|n)=n\varphi(\varsigma|n).$$

The operator $\hat{n}=\hat{\alpha}^{+}\hat{\alpha}$ denomination is operator number. With its help the photons operator number will be constructed. If the wave function $\psi(\varsigma)$ is not the same with any $\varphi(\varsigma|n)$, the number of photons in such state have not a specific meaning, and one can only talk about their average quantum number $\langle\hat{n}\rangle=\int_{-\infty}^{\infty}\psi^{*}(\varsigma)\hat{n}\psi(\varsigma)d\varsigma$. In General, if the oscillator is acting by externally force, then the wave function can essentially depends on time. Instead of equation (5), its behaviour is now defines a temporary Schrödinger equation:

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=\hat{H}\Psi(x,t).$$

3. THE PROCEDURE OF QUANTIZATION OF THE ELECTROMAGNETIC FIELD

We will say that some set of parameters describes the state of any material object in some moment of time $t=0$, if this set is sufficient to the prediction of the results of any potential experiment carried out on this object in a future time $t>0$. In classical mechanics the state of a point particle in the one-dimensional space is described by coordinate x and momentum p. In the transition from classical mechanics to the quantum one the coordinate of a particle and the momentum are replaced by the corresponding operators. A similar situation arises in the field theory. In classical theory of electromagnetic field the state of the field is determines by its amplitude $A^{\nu}(\boldsymbol{r},t)$. It means that by the transition from classical optics to quantum one the field amplitude has to be replaced with an operator expression. The state of any system in quantum theory describes the wave function ψ satisfying to Schrödinger equation. The explicit form of the Hamiltonian in this equation will be predicted by procedure of quantization of classical oscillator with the replacement of electromagnetic energy by its quantum analogue. In classical physics, the energy of plane electromagnetic waves occupying the volume V is described by formula:

$$E=\int_V\frac{E^2(\boldsymbol{r},t)}{8\pi}d\boldsymbol{r}+\int_V\frac{H^2(\boldsymbol{r},t)}{8\pi}d\mathbf{r}=\int_V\frac{E^2(\boldsymbol{r},t)}{4\pi}d\boldsymbol{r}.$$

Here is taken into account that at plane electromagnetic waves the energy falling on the electric and magnetic components is equal to each other. Let us assume that electromagnetic field is placed in a cube with edge L and volume $V=L^3$. At the borders of the cube one use the periodic boundary conditions $\exp(ik_xL)=\exp(ik_yL)=\exp(ik_zL)=1$. In the case, using (2) and (3), we obtain

$$E=\sum_{\mathbf{k}\lambda}\frac{V\omega_k^2}{4\pi c^2}\gamma_k^2\left(\alpha_{\mathbf{k}\lambda}^{*}\alpha_{\mathbf{k}\lambda}+\alpha_{\mathbf{k}\lambda}\alpha_{\mathbf{k}\lambda}^{*}\right),\ \omega_k=ck. \qquad (9)$$

This form of energy emphasizes its reality. The transition to infinite volume carries out by procedure $V\to\infty$. Comparison of expressions (6) and (9) indicates that the quantization procedure demands the replacement

$$\gamma_k\to\sqrt{\frac{2\pi\hbar c^2}{\omega_kV}},\ \alpha_{\mathbf{k}\lambda}\to\hat{\alpha}_{\mathbf{k}\lambda}=\frac{1}{\sqrt{2}}\left(\varsigma_{\mathbf{k}\lambda}+\frac{\partial}{\partial\varsigma_{\mathbf{k}\lambda}}\right),$$
$$\alpha_{\mathbf{k}\lambda}^{*}\to\hat{\alpha}_{\mathbf{k}\lambda}^{+}=\frac{1}{\sqrt{2}}\left(\varsigma_{\mathbf{k}\lambda}-\frac{\partial}{\partial\varsigma_{\mathbf{k}\lambda}}\right). \qquad (10)$$

At the same time, vector potential (3) and total field energy (9) are replaced by operator expressions

$$\hat{A}^{\nu}(\boldsymbol{r},t)=\sum_{\mathbf{k}\lambda}e_{\mathbf{k}\lambda}^{\nu}\gamma_k\left(\hat{\alpha}_{\mathbf{k}\lambda}^{i\mathbf{kr}-ikct}+\hat{\alpha}_{\mathbf{k}\lambda}^{+}e^{-i\mathbf{kr}+ikct}\right),$$
$$\hat{H}=\sum_{\mathbf{k}\lambda}\frac{\hbar\omega_k}{2}\left(\hat{\alpha}_{\mathbf{k}\lambda}^{+}\hat{\alpha}_{\mathbf{k}\lambda}+\hat{\alpha}_{\mathbf{k}\lambda}\hat{\alpha}_{\mathbf{k}\lambda}^{+}\right). \qquad (11)$$

View of the Schrödinger equation containing this operator $\hat{H}$ (Hamiltonian) was given above

$$\hat{H}\psi(\mathrm{V}) = E\psi(\mathrm{V}).$$

The solution to this equation is the product of functions

$$\varphi(\varsigma|\mathbf{N}) = \prod_{\mathbf{k}\lambda}\varphi(\varsigma_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda}),\ E_{\mathbf{k}\lambda} = \hbar\omega_k\left(n_{\mathbf{k}\lambda} + \frac{1}{2}\right),$$
$$E = \sum_{\mathbf{k}\lambda} E_{\mathbf{k}\lambda}.$$

Here, the multidimensional vector ς means a set of arguments ....,$\varsigma_{\mathbf{k}\lambda}$,... with varyous $(\mathbf{k},\lambda)$, the multidimensional vector $\mathbf{N}$ means a set of numbers ...,$n_{\mathbf{k}\lambda}$,.... Function $\varphi(\varsigma_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda})$ satisfies the equation

$$\frac{\hbar\omega_k}{2}\left(\hat{\alpha}^+_{\mathbf{k}\lambda}\hat{\alpha}_{\mathbf{k}\lambda} + \hat{\alpha}_{\mathbf{k}\lambda}\hat{\alpha}^+_{\mathbf{k}\lambda}\right)\varphi(\varphi_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda}) = E_{\mathbf{k}\lambda}\varphi(\varphi_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda}).$$

The energy $E_{\mathbf{k}\lambda} = \hbar\omega_k(n_{\mathbf{k}\lambda} + 1/2)$ corresponds to each solution φ $(\varphi_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda})$. Since we are always interested in energy difference

$$E - \sum_{\mathbf{k}\lambda}\hbar\omega_k/2 = \sum_{\mathbf{k}\lambda}\hbar\omega_k n_{\mathbf{k}\lambda}, \qquad (12)$$

then on the amount $\sum_{\mathbf{k}\lambda}\hbar\omega_k/2$ one could not pay attention. The sum in the left side of equality (12) is energy full electromagnetic fields associated with the specific set of numbers $n_{\mathbf{k}\lambda}$ that is, a specific set of photons, each with energy $\hbar\omega_k$. Thus $n_{\mathbf{k}\lambda}$ means number of photons in the mode $(\mathbf{k},\lambda)$.

So, the photon $(n_{\mathbf{k}\lambda} = 1)$ of the mode $(\mathbf{k},\lambda)$ is the state of the electromagnetic field to which corresponds the wave function

$$\varphi(\varsigma_{\mathbf{k}\lambda}|1) = H_1(\mathrm{V}_{\mathbf{k}\lambda})\exp(-\varsigma^2_{\mathbf{k}\lambda}/2) = \sqrt{2}\pi^{-1/4}\varsigma_{\mathbf{k}\lambda}\exp(-\varsigma^2_{\mathbf{k}\lambda}/2). \qquad (13)$$

This is the answer to the question in the title of this article. In infinite space, such states in its pure form cannot exist, as well as in the classical physics strictly monochromatic waves cannot exist. It's impossible just because monochromatic waves have no boundaries. But researching their properties is extremely fruitful, as soon as any really existing electromagnetic field can be represented in the form of their superposition. The same property is possessed by the photon states. Any realistically state of the field can be built of them, as from bricks.

To understand the physical meaning of a function (13), it is necessary to find out the physical meaning of arguments $\varsigma_{\mathbf{k}\lambda}$. The procedure of quantization of fields in accordance with (10) demands the replacement

$$\frac{\alpha_{\mathbf{k}\lambda} + \alpha^*_{\mathbf{k}\lambda}}{\sqrt{2}} = \frac{A_{\mathbf{k}\lambda}}{\gamma_k\sqrt{2}}\cos\vartheta_{\mathbf{k}\lambda} \to \varsigma_{\mathbf{k}\lambda}$$
$$\frac{\alpha_{\mathbf{k}\lambda} - \alpha^*_{\mathbf{k}\lambda}}{\sqrt{2}} = i\frac{A_{\mathbf{k}\lambda}}{\gamma_k\sqrt{2}}\sin\vartheta_{\mathbf{k}\lambda} \to \frac{\partial}{\partial\varsigma_{\mathbf{k}\lambda}}. \qquad (14)$$

The argument of the wave function $\varsigma_{\mathbf{k}\lambda}$ corresponds to "classical value" $A_{\mathbf{k}\lambda}\cos\vartheta_{\mathbf{k}\lambda}/\gamma_k\sqrt{2}$. The word "classical" we take in inverted commas, because γ_k contents $\hbar$. Of course, it would be possible to write in such form $\varphi(A_{\mathbf{k}\lambda}\cos\vartheta_{\mathbf{k}\lambda}/\gamma_k\sqrt{2}|n_{\mathbf{k}\lambda})$, but this is inconvenient, so one is writing $\varphi(\varsigma_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda})$. According to quantum principles, construction $|\varphi(\varsigma_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda})|^2 d\varsigma_{\mathbf{k}\lambda}$ determines the probability about detection argument $\varsigma_{\mathbf{k}\lambda}$ (or "classical" construction $A_{\mathbf{k}\lambda}\cos\vartheta_{\mathbf{k}\lambda}/\gamma_k\sqrt{2}$) in the interval $d\varsigma_{\mathbf{k}\lambda}$, if such experiment will be delivered. In such a way one founds the distribution function of the "classical" variable $A_{\mathbf{k}\lambda}\cos\vartheta_{\mathbf{k}\lambda}/\gamma_k\sqrt{2}$.

We consider the Fourier transform (indices **k**, λ are omitted)

$$\varphi(\varsigma) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{i\eta\varsigma}\tilde{\varphi}(\eta)d\eta,$$
$$\tilde{\varphi}(\eta) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-i\eta\varsigma}\varphi(\varsigma)d\varsigma.$$

From the properties of the Fourier transformation it follows that this procedure is accompanied by replacement

$$\varphi(\varsigma) \leftrightarrow \tilde{\varphi}(\eta),\ \varsigma \leftrightarrow i\frac{\partial}{\partial\eta},\ -i\frac{\partial}{\partial\varsigma} \leftrightarrow \eta$$

Therefore, instead of (14) as quantization rules of the electromagnetic fields can serve as the following:

$$\frac{\alpha_{\mathbf{k}\lambda}+\alpha^*_{\mathbf{k}\lambda}}{\sqrt{2}}=\frac{A_{\mathbf{k}\lambda}}{\gamma_k\sqrt{2}}\cos\vartheta_{\mathbf{k}\lambda}\to i\frac{\partial}{\partial\eta_{\mathbf{k}\lambda}},$$
$$\frac{\alpha_{\mathbf{k}\lambda}-\alpha^*_{\mathbf{k}\lambda}}{\sqrt{2}}=i\frac{A_{\mathbf{k}\lambda}}{\gamma_k\sqrt{2}}\sin\vartheta_{\mathbf{k}\lambda}\to i\eta_{\mathbf{k}\lambda}. \quad (15)$$

Thus, the argument of the function $\tilde{\varphi}(\eta_{\mathbf{k}\lambda})$ meets the value of the classic $A_{\mathbf{k}\lambda}\sin\vartheta_{\mathbf{k}\lambda}/\gamma_k\sqrt{2}$. Construction $|\tilde{\varphi}(\eta_{\mathbf{k}\lambda})|^2$ determines the probability distribution "classical" values $A_{\mathbf{k}\lambda}\sin\vartheta_{\mathbf{k}\lambda}/\gamma_k\sqrt{2}$. Because the function $\tilde{\varphi}(\eta_{\mathbf{k}\lambda})$ follows from $\varphi(\varsigma_{\mathbf{k}\lambda})$ by a Fourier transformation, then according to (13)

$$\tilde{\varphi}(\eta_{\mathbf{k}\lambda}|1)=i\sqrt{2}\pi^{-1/4}\eta_{\mathbf{k}\lambda}\exp(-\eta^2_{\mathbf{k}\lambda}/2).$$

So, for one-photon states we have:

$$|\varphi(\varsigma_{\mathbf{k}\lambda}|1)|^2=\frac{2}{\sqrt{\pi}}\varsigma^2_{\mathbf{k}\lambda}\exp(-\varsigma^2_{\mathbf{k}\lambda}),$$
$$|\tilde{\varphi}(\eta_{\mathbf{k}\lambda}|1)|^2=\frac{2}{\sqrt{\pi}}\eta^2_{\mathbf{k}\lambda}\exp(-\eta^2_{\mathbf{k}\lambda}).$$

These are even functions of their arguments, so quantum averages

$$\langle\varsigma_{\mathbf{k}\lambda}\rangle=\langle\eta_{\mathbf{k}\lambda}\rangle=\int_{-\infty}^{\infty}\varsigma_{\mathbf{k}\lambda}|\varphi(\varsigma_{\mathbf{k}\lambda})|^2d\varsigma_{\mathbf{k}\lambda}=$$
$$=\int_{-\infty}^{\infty}\eta_{\mathbf{k}\lambda}|\tilde{\varphi}(\eta_{\mathbf{k}\lambda})|^2d\eta_{\mathbf{k}\lambda}=0$$

turned to zero. In other words, in one-photon states the quantum average $\langle A_{\mathbf{k}\lambda}\cos\vartheta_{\mathbf{k}\lambda}\rangle=\langle A_{\mathbf{k}\lambda}\sin\vartheta_{\mathbf{k}\lambda}\rangle=0$ is vanishing. For this reason, the classical parameters $A_{\mathbf{k}\lambda}$ and $\vartheta_{\mathbf{k}\lambda}$ cannot describe the single-photon states. Let's emphasize separately and once again that the single-photon state of the electromagnetic field do not has such attributes as amplitude $A_{\mathbf{k}\lambda}$ and phase $\vartheta_{\mathbf{k}\lambda}$.

But such property is inherent not only to one-photon states, but also to any states with precisely defined number of photons (Fock states). Really, after averaging over any defined by function $\varphi(\varsigma_{\mathbf{k}\lambda}|n_{\mathbf{k}\lambda})$ the quantum average of a vector potential (11)

$$\langle\hat{A}^\nu(\mathrm{r},t)\rangle=$$
$$=\sum_{\mathbf{k}\lambda}e^\nu_{\mathbf{k}\lambda}\gamma_k\left(\langle\hat{\alpha}_{\mathbf{k}\lambda}\rangle e^{i\mathbf{kr}-ikct}+\langle\hat{\alpha}^+_{\mathbf{k}\lambda}\rangle e^{-i\mathbf{kr}+ikct}\right)=0$$

turns to zero, which follows from the relations (8). But the quantum average $\langle\hat{A}^\nu(\mathbf{r},t)\rangle$ cannot be vanish, if we are dealing with superposition of wave functions with different numbers $n_{\mathbf{k}\lambda}$. It is among these superposition's necessary to find the wave function describing the quantum state that most closely approximates a classic field. Such a superposition is (again, we omit the indices $\mathbf{k},\lambda$):

$$\varphi(\varsigma|\alpha)=e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\varphi(\varsigma|n),$$
$$\int_{-\infty}^{\infty}\varphi^*(\varsigma|\alpha)\varphi(\varsigma|\alpha)d\varsigma=1,$$

where α is any number. It is customary to speak, that this superposition describes the "quantum coherent" state. It is easy to see that $\hat{\alpha}\varphi(\varsigma|\alpha)=\alpha\varphi(\varsigma|\alpha)$. If the operator $\hat{\alpha}$ to describe in explicit form (10), we get the equation $\frac{1}{\sqrt{2}}\left(\varsigma+\frac{\partial}{\partial\varsigma}\right)\varphi(\varsigma|\alpha)=\alpha\varphi(\varsigma|\alpha)$, the solution of which is a function

$$\varphi(\varsigma|\alpha)=\pi^{-1/4}\exp\left[-\frac{(\alpha+\alpha^*)^2}{4}+\sqrt{2}\alpha\varsigma-\frac{1}{2}\varsigma^2\right]. \quad (16)$$

When working with quantum coherent states the following integral is useful:

$$\int_{-\infty}^{\infty}\exp(\delta\varsigma-\beta\varsigma^2)d\varsigma=\sqrt{\frac{\pi}{\beta}}\exp\left(\frac{\delta^2}{4\beta}\right).$$

Now, it is clear that $\int_{-\infty}^{\infty}\varphi^*(\varsigma|\alpha)\varphi(\varsigma|\alpha)d\varsigma=1$.

We use the Fourier transform

$$\tilde{\varphi}(\eta|\alpha)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-i\eta\varsigma}\varphi(\varsigma|\alpha)d\varsigma=$$

$$=\pi^{-1/4}\exp\left[-\frac{\left(\alpha+\alpha^*\right)^2}{4}+\alpha^2-i\sqrt{2}\alpha\eta-\frac{1}{2}\eta^2\right].$$

It follows from (16) and (17) that

$$\left|\varphi(\varsigma|\alpha)\right|^2=\frac{1}{\sqrt{\pi}}\exp\left[-\left(\varsigma-\frac{\alpha+\alpha^*}{\sqrt{2}}\right)^2\right],$$
$$\left|\hat{\varphi}(\eta|\alpha)\right|^2=\frac{1}{\sqrt{\pi}}\exp\left[-\left(\eta-\frac{\alpha-\alpha^*}{i\sqrt{2}}\right)^2\right]. \quad (18)$$

If one use the ideas $\alpha=|\alpha|\exp(i\vartheta)$ and $\alpha^*=|\alpha|\exp(-i\vartheta)$, then the most probable values of ς_{ex} and η_{ex} according to (14) and (15) will be:

$$\varsigma_{ex}=\left(A\cos\vartheta/\gamma_k\sqrt{2}\right)_{ex}=\frac{\alpha+\alpha^*}{\sqrt{2}}=\sqrt{2}|\alpha|\cos\vartheta,$$
$$\eta_{ex}=\left(A\sin\vartheta/\gamma_k\sqrt{2}\right)_{ex}=\frac{\alpha-\alpha^*}{i\sqrt{2}}=\sqrt{2}|\alpha|\sin\vartheta.$$

At $2\gamma_k|\alpha|=A$ from these ratios, the expressions followed which are valid for classical waves.

Next, we need an auxiliary equality

$$\langle n\rangle=\int_{-\infty}^{\infty}\varphi^*(\varsigma|\alpha)\hat{\alpha}^+\hat{\alpha}\varphi(\varsigma|\alpha)d\varsigma=$$
$$=\alpha\int_{-\infty}^{\infty}\varphi^*(\varsigma|\alpha)\hat{\alpha}^+\varphi(\varsigma|\alpha)d\varsigma=$$
$$=\alpha\int_{-\infty}^{\infty}\hat{\alpha}\varphi^*(\varsigma|\alpha)\,\varphi(\varsigma|\alpha)d\varsigma=\alpha\alpha^*.$$

Because the averaging on coherent states gets

$$\left\langle\hat{A}^\nu(\mathbf{r},t)\right\rangle=\sum_{\mathbf{k}\lambda}e^\nu_{\mathbf{k}\lambda}\gamma_k\left(\alpha_{\mathbf{k}\lambda}e^{i\mathbf{kr}-ikct}+\alpha^*_{\mathbf{k}\lambda}e^{-i\mathbf{kr}+ikct}\right),$$
$$\left\langle E^\nu(r,t)\right\rangle=-\frac{1}{c}\frac{\partial}{\partial t}\left\langle E^\nu(r,t)\right\rangle=$$
$$=\sum_{k\lambda}e^\nu_{k\lambda}E_{k\lambda}\sin(kr-kct+\vartheta)\ ,$$
$$E_{\mathbf{k}\lambda}=-2\sqrt{\frac{2\pi\hbar\omega_k}{V}}|\alpha_{\mathbf{k}\lambda}|,$$

so for these states, taking into account the auxiliary equality written in the form $|\alpha|=\sqrt{\langle n\rangle}$, one is finding $E^2_{\mathbf{k}\lambda}V/8\pi=\hbar\omega_k\langle n_{\mathbf{k}\lambda}\rangle$. Introducing the concept of effective amplitude of the field $\bar{E}_{\mathbf{k}\lambda}=E_{\mathbf{k}\lambda}/\sqrt{2}$, we conclude $\bar{E}^2_{\mathbf{k}\lambda}V/4\pi=\hbar\omega_k\langle n_{\mathbf{k}\lambda}\rangle$. So, if the electromagnetic wave is in a quantum coherent state, it does not possess a fixed number of photons, but its energy can be calculated as a quantum formula and classical one.

A quantum averages and the most probable parameters of the electromagnetic field coincide with their classical analogues, if the free field is in quantum coherent state. But that doesn't mean that each experiment will give the values of classical parameters. According to the distributions (18), there will be dispersion of points. The magnitude of this dispersion is determined by the dispersions of the distributions (18), which do not depend on the amplitudes of the fields. For this reason, if the amplitudes are large, then the dispersions can be neglected.

In this case, a flat electromagnetic wave, finding in the quantum coherent state, is described well by the classical theory. But that doesn't mean, that the result of the interaction of such waves with the environment always admits a classical description. Such example is set out below. Once again, emphasize that large value of the quantum coherent field by itself is not enough to use classical Maxwell equation.

It is useful to notice that in Fock state standing field with a large number of photons can to give a very high energy and destructive force, but the classical description of such a field does not exist.

4. THE INTERACTION OF THE ELECTROMAGNETIC FIELD WITH THE ENVIRONMENT

We will discuss examples showing that the evolution as strong as one wants and being in the quantum coherent state $\left\langle\hat{A}(\mathbf{r},t)\right\rangle\neq 0$ electromagnetic fields may not obey the laws of classical physics.

Consider a system consisting of excited atom and quantized electromagnetic field interacting each to another. For the sake of simplicity, we suppose, that the atom has one valence electron and two energy levels: ground and excited. The wave function of an electron in an excited atom denote by $\psi_{j_{ex}}(\boldsymbol{r})$, in the ground state via $\psi_{j_g}(\boldsymbol{r})$. Spin effects will neglect.

Let the atom, placed in an excited state-$\psi_{j_{ex}}(\boldsymbol{r})$ undergoes action of the single –mode $(\mathbf{k}_0\lambda_0)$ radiation, placed in a quantum coherent state $\varphi(\varsigma\,|\alpha)$. As a result of the scattering process, the total wave function atom-field system takes the form

$$\Psi(\mathrm{t}) = f_{j_{ex}}(\mathrm{t})\psi_{j_{ex}}(\boldsymbol{r}) + f_{j_g}(\mathrm{t})\psi_{j_g}(\boldsymbol{r}). \qquad (19)$$

Function $f_{ex}(\mathrm{t})$ describes the coherent channel of scattering, as a result of which the scattering atom remains in the initial excited state. Function $f_g(\mathrm{t})$ describes the scattering channel, as a result of which the atom of changes its state on ground state. This channel will be called incoherent. Since in the initial state electromagnetic field has "quantum average" value of the amplitude $\left\langle \hat{A}^{\nu}(\boldsymbol{r},t)\right\rangle$, then after scattering this construction will not turn to zero. Below, instead of operator $\hat{A}^{\nu}(\boldsymbol{r},t)$ one will use the operator of tension of the electric field

$$\hat{E}^{\nu}(k,t) = -\frac{1}{c}\frac{\partial\hat{A}^{\nu}(\boldsymbol{r},t)}{\partial t} =$$
$$= i\sum_{\mathbf{k}\lambda} e^{\nu}_{\mathbf{k}\lambda} k\gamma_k\left(\hat{\alpha}_{\mathbf{k}\lambda}{}^{i\mathbf{kr}-ikct} - \hat{\alpha}^{+}_{\mathbf{k}\lambda}e^{-i\mathbf{kr}+ikct}\right).$$

"Quantum average" of electric tension of electromagnetic field after scattering is calculating by conventional rule

$$\left\langle \hat{E}^{\nu}(\boldsymbol{r},t)\right\rangle = \left\langle \Psi\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\middle|\Psi\right\rangle =$$
$$= \left\langle f_{ex}\psi_{j_{ex}}\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\middle|f_{ex}\psi_{j_{ex}}\right\rangle +$$
$$+\left\langle f_g\psi_{j_g}\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\middle|f_g\psi_{j_g}\right\rangle =$$
$$= \left\langle f_{ex}\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\middle|f_{ex}\right\rangle + \left\langle f_g\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\middle|f_g\right\rangle =$$
$$= E^{\nu(\mathrm{c})}(\boldsymbol{r},t) + E^{\nu(\mathrm{n})}(\boldsymbol{r},t). \qquad (20)$$

Here in explicit form the function $\Psi(\mathrm{t})$ describing by expression (19) is shown, with the help of which the quantum averaging is performed, and it is taken into account that the operator $\hat{E}^{\nu}(\boldsymbol{r},t)$ does not change the mutually orthogonal $\left\langle\psi_j\middle|\psi_{j'}\right\rangle = \int\psi^*_j(\boldsymbol{r})\psi_{j'}(\boldsymbol{r}) = \delta_{jj'}$ and normalized per unit the wave atoms functions. For these reasons, atomic functions fall out of this formulas, as well as fall out the interference term

$$\left\langle f_{ex}\psi_{j_{ex}}\middle|\hat{A}^{\nu}(\boldsymbol{r},t)\middle|f_g\psi_{j_g}\right\rangle = 0.$$

It is important to note that interference term turns to zero because of orthogonality of atomic functions, which eliminates the "quantum interference terms" of the electromagnetic field. The classical physics does not possess such property. Let's to say that coherent and incoherent channels of scattering are not quantum incoherent. But classical coherence in the form of summation of amplitudes with according to (20) is retained. From the coherent scattering channel the process of stimulated radiation that changes the state of the scattering atoms fall out. Thus, it should be distinguished two types of coherence: "quantum" depending on phase of full wave function of the system, and "classical", determined by the phase of averaged amplitude of the field. At the process of averaging the quantum phases disappear.

Suppose now we are interested in the energy characteristics of the scattered field, describing the "quantum averages" from the bilinear combinations of the field operators

$$\left\langle\hat{E}^{\nu}(\boldsymbol{r},t)\hat{E}^{\nu}(\boldsymbol{r},t)\right\rangle = \left\langle\Psi\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\hat{E}^{\nu}(\boldsymbol{r},t)\middle|\Psi\right\rangle =$$
$$= \left\langle f_{ex}\psi_{j_{ex}}\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\hat{E}^{\nu}(\boldsymbol{r},t)\middle|f_{ex}\psi_{j_{ex}}\right\rangle + \qquad (21)$$
$$+\left\langle f_g\psi_{j_g}\middle|\hat{E}^{\nu}(\boldsymbol{r},t)\hat{E}^{\nu}(\boldsymbol{r},t)\middle|f_g\psi_{j_g}\right\rangle.$$

Here again and for the same reason there is no cross term. Both terms are positively determined.

Thus, the energy characteristics of the scattered field are determined independently by two channels of reactions and then algebraically add up. According to (21) the interference between channels is absent. Very important is to note that there is no interference between placed in the incoherent channel processes of induced radiation and scattering processes that do not change states of the atom system. In this sense, incoherent properties of induced processes, which rarely stressed, can significantly change the macroscopic pattern of the scattered field.

The value of the averaged bilinear structures (21) it is convenient to estimate using the well-known inequality

$$\left\langle \hat{B}\hat{B} \right\rangle \gg \left\langle \hat{B}\hat{B} \right\rangle \left\langle \hat{B}\hat{B} \right\rangle,$$

true for any operators and any procedure of averaging. Applying this inequality to both terms equality (21), we obtain the lower estimate for the research construction

$$\left\langle \hat{E}^{\nu}(\boldsymbol{r},t)\hat{E}^{\nu}(\boldsymbol{r},t) \right\rangle \gg \\ \gg E^{\nu(\mathrm{c})}(\boldsymbol{r},t)E^{\nu(\mathrm{c})}(\boldsymbol{r},t) + E^{\nu(\mathrm{n})}(\boldsymbol{r},t)E^{\nu(\mathrm{n})}(\boldsymbol{r},t). \quad (22)$$

If only the non-coherent channel (and the processes of induced radiation) are absent or on any reason the coherent channel of scattering is absent, the energy of the scattered electromagnetic field is determined by the squared of scattering amplitude, which resemble equality, at in classic field. In other words, the classical physics can correctly describe the scattering process of resonant radiation only in exceptional cases. In general, if the scattering radiation is almost classical and is in a quantum coherent state the scattered radiation loses this property. According to (22), the energy of the scattered radiation consists of two terms that are defined by different channels of scattering. Energy characteristics of these channels summed. “Quantum coherence states” do not obey such properties. One can say that in the scattered radiation, along the quantum coherent component the “Fock’s” component appears. The value of “Fock’s” component can reach rather hundred percents of the total radiation. Such radiation in the representations of classical physics can’t be described. This is the case with Fresnel reflection of resonance radiation from excited media [6]. They say that in these cases we have dealing with the discrepancy between the predictions of the classical and quantum physics at a macroscopic level.

5. CONCLUSION

Pay attention to the equality

$$\varphi(\varsigma|\alpha) = \pi^{-1/4}\exp\left[-\frac{\left(\alpha+\alpha^*\right)^2}{4} + \sqrt{2}\alpha\varsigma - \frac{1}{2}\varsigma^2\right] = \\ = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\varphi(\varsigma|n),$$

which is indicating that the closest to the classical presentation the concepts of “quantum coherence states” of electromagnetic field can be represented as a sum of photons (Fock states) states having no classical analogues. It is a “quantum coherent state” $\varphi(\varsigma|\alpha)$can be described verbally with the help of concepts “amplitude”, “phase” and “dispersions” of these quantities, then as photons (Fock) states $\varphi(\varsigma|n)$ of these attributes lack. In Fock states we use the adoption of a “photon”. There is also an inverse relationship [7]:

$$\varphi(\varsigma|n) = \frac{1}{\pi}\iint\frac{\alpha^{*n}}{\sqrt{n!}}e^{-\frac{1}{2}|\alpha|^2}\varphi(\varsigma|\alpha)d^2\alpha, \\ d^2\alpha = d(\mathrm{Re}\,\alpha)d(\mathrm{Im}\,\alpha),$$

which is indicating that photons (Fock) states, incognizant the concepts of “phase” and “amplitude”, can be present in the form of a superposition of “quantum coherent states”, which are characterized by such concepts. One can speak that we solve the problem in photon (Fock) or in a coherent representation. Therefore, the objective characteristic of the electromagnetic field is its state described by the wave function. For depending on the mathematical background chosen by us, the specific view of this function can be strongly various. It may be representing by the series of functions $\varphi(\varsigma|n)$ as well as functions $\varphi(\varsigma|n)$.

The attributes such as amplitude, phase, and number of photons, are inherent to concrete representations and are not invariant with respect to the choice that depends on us. Thus, these attributes do not follow absolutely sense and do not possess universal meaning.

The need for quantization of the electromagnetic field follows from logical considerations and from different results of calculations using quantized and classical fields that qualitatively demonstrated above on some examples.

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