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ON VARIANTS OF THE MAIN ATMOSPHERIC CORRECTION FORMULA

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ABSTRACT

The article studies the accuracy of the main formula of atmospheric correction allowing us to determine albedo of a underlying (earth) surface based on radiance factor of solar light reflected by the system of atmosphere and underlying surface. The problem of atmospheric correction is considered in three-dimensional geometry with spatial non-uniformity of the underlying surface taken into account. It is demonstrated that the accuracy of albedo recovery depends on the used variant of the main formula.

Keywords: atmospheric correction, radiance factor, surface albedo, multidimensional effects

1. INTRODUCTION

During Earth remote sensing, the values of intensity (raiance) of solar radiation (SR) reflected by the system of atmosphere and underlying surface (US) are measured. The measured values of intensity and the known values of extraatmospheric intensity of SR allow one to calculate the radiance factor (RF). Atmospheric correction is required to remove atmospheric distortions from the radiance factor and to determine the reflectance of the Earth surface (albedo). The obtained values of albedo may be further used to determine the surface composition and properties.

In atmospheric correction problems, atmosphere is usually assumed to be horizontally homogeneous and the surface is assumed to be heterogeneous. The surface and the upper border of the atmosphere are divided into pixels. The main atmospheric correction formula relates albedo in Earth pixels and values of RF in atmospheric pixels.

Originally, the main formula was found on the basis of assumption that light gets into an atmospheric pixel only from the Earth pixel located right underneath it. In other words, the problem was solved in Independent Pixel Approximation (IPA). And SR intensity in the problem with random value of the surface albedo was represented by a combination of intensity of light from a black (non-reflecting) US and from a red (isotropic emitting light) US [1]. The formula allows us to determine exactly the US albedo based on the value of RF. This formula is actively used in atmospheric correction problems [2].

With increase of spatial resolution, it became necessary to take into account what contribution is made by light reflected from each Earth pixel to the signal registered in each atmospheric pixel. Two methods are used here.

The first method is based on empirical generalisation of the IPA model, in which RF in an atmospheric pixel depends on albedo in the corresponding Earth pixel and albedo of surroundings of this pixel [3]. It is possible to use an explicit and thus fast algorithm, in which first all surrounding albedos are found and then albedos of all Earth pixels are found [3–5].

In the second method, a system of nonlinear equations is built considering not only reflection of light from an earth's pixel to an atmospheric one but also re-reflection between earth's pixels [4]; unknown variables are albedos of all earth's pixels. The process of solving such system of equation is extremely time-consuming. Required time may be slightly reduced by simplifying the system of equations by excluding re-reflection between earth's pixels located far from each other. For atmospheric correction in mountainous regions during modelling of re-reflection between earth's pixels, terrain should be taken into account [5].

In [6], the formula of atmospheric correction explicitly relating albedo in each Earth pixel with RF in all atmospheric pixels is proposed. The formula allows one to quickly find albedo in each pixel with consideration of re-reflection from other pixels [7]. The formula is based on representation of SR intensity in the task with random value of albedo as a linear combination of intensities in the problem with black US and in problems with US having one white pixel (reflecting the entire radiation in accordance with the Lambert law) and remainder black pixels.

This work proposes another explicit formula using the conventional linear combination of intensities in the problem with black surface and in problems with one red pixel and remaining black pixels. The matter of identity of these formulas and their accuracies for recovery of albedo of a spatially non-uniform surface is also considered.

2. THE PROBLEM OF LIGHT TRANSFER IN THE ATMOSPHERE

Let us consider the problem of transfer of monochromatic light in a three-dimensional area, Fig. 1.

$$\hat{T}^{3D} I = \mu \frac{\partial I}{\partial z} + \xi \frac{\partial I}{\partial x} + \eta \frac{\partial I}{\partial y} + \\ + k(x,y,z)\, I(x,y,z,\mu,\varphi) - \\ - k_s(x,y,z) \int_{\Omega} P(x,y,z,\gamma_s(\mu,\mu',\varphi,\varphi')) \\ I(x,y,z,\mu',\varphi')\, d\mu'\, d\varphi' = 0 \quad , \qquad (1)$$

$$-X < x < X,\ \ -Y < y < Y,\ \ 0 < z < H, \\ -1 < \mu < 1,\ \ 0 < \varphi < 2\pi, \qquad (2)$$

$$\xi = \sin\theta\cos\varphi,\ \eta = \sin\theta\sin\varphi,\ \mu = \cos\theta, \\ \gamma_s(\mu,\mu',\varphi,\varphi') = \mu\mu' + \\ + \sqrt{1-\mu^2}\,\sqrt{1-(\mu')^2}\,\cos(\varphi-\varphi'). \qquad (3)$$

The solution $I(x,y,z,\mu,\varphi)$ here is the SR intensity at a spatial point with coordinates (x,y,z) in the $\Omega(\mu=\cos\theta,\varphi)$ direction, Fig. 1. The extinction factor $k(x,y,z)$, the scattering factor $k_s(x,y,z)$, and the scattering indicatrix $P(x,y,z,\gamma_s)$ generally depend both on the height z and horizontal coordinates x, y. The scattering indicatrix depends on $\gamma_s(\mu,\mu',\varphi,\varphi')$, cosine of the angle between the directions (μ,φ) and (μ',φ').

At the upper border, z=0, of the three-dimensional area $-X < x < X$, $-Y < y < Y$, $0 < z < H$, Fig. 1, the condition of incident parallel beam of SR rays in the direction $\Omega_0(\mu_0 = \cos\theta_0, \varphi_0)$ is set:

$$I(x,y,0,\mu,\varphi) = I_0\,\delta(\mu-\mu_0)\,\delta(\varphi-\varphi_0) \\ \text{at } \mu > 0,\ -X < x < X,\ \ -Y < y < Y. \qquad (4)$$

Here I_0 is the extraatmospheric SR and δ is the Dirac delta function.

At the lower border $z = H$, let us set reflection from the surface according to the Lambert law

$$I(x,y,H,\mu,\varphi) = \\ = A(x,y)\,\frac{1}{\pi}\int_0^1 d\mu'\,\mu' \int_0^{2\pi} d\varphi'\, I(x,y,H,\mu',\varphi')$$

at $\mu < 0$, $-X < x < X, -Y < y < Y$, (5)

where $A(x,y)$ is the albedo of the US at the point (x,y). We will assume that the US is divided into N non-intersecting pixels U_j. Let us set surface albedo within each pixel with its mean value:

$$A_j = \frac{1}{|U_j|}\iint_{U_j} dx\,dy\,A(x,y), \qquad (6)$$

where $|U_j|$ is the area of the j-th pixel. At irradiated side borders where $\gamma_s(\mu,\mu_0,\varphi,\varphi_0) > 0$, the bound-

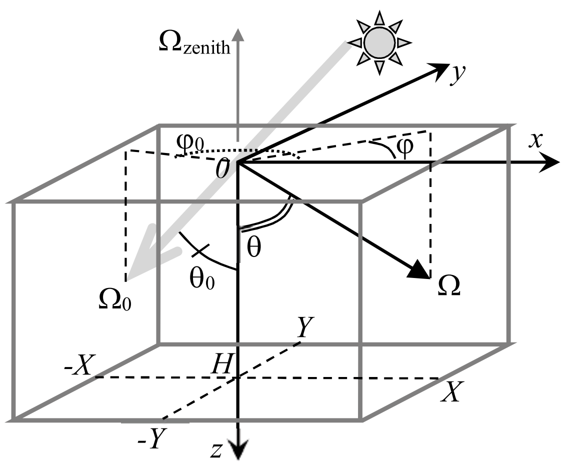

Fig. 1. Area of the solution of the solar light transfer equation

ary condition in the form of (4) is set. At non-irradiated side borders where $\gamma_s(\mu,\mu_0,\varphi,\varphi_0)<0$, zero boundary condition is set. Thus, we have the relations:

$$I(-X,y,z,\mu,\varphi)=J(\mu,\varphi) \quad \text{at } \xi>0,$$

$$-Y<y<Y,\quad 0<z<H, \tag{7}$$

$$I(X,y,z,\mu,\varphi)=J(\mu,\varphi) \quad \text{at } \xi<0,$$

$$-Y<y<Y,\quad 0<z<H, \tag{8}$$

$$I(x,-Y,z,\mu,\varphi)=J(\mu,\varphi) \quad \text{at } \xi>0,$$

$$-X<x<X,\quad 0<z<H, \tag{9}$$

$$I(x,Y,z,\mu,\varphi)=J(\mu,\varphi) \quad \text{at } \xi<0,$$

$$-X<x<X,\quad 0<z<H, \tag{10}$$

where

$$J(\mu,\varphi)=I_0\,\delta(\mu-\mu_0)\delta(\varphi-\varphi_0)$$
$$\text{при } \gamma_s(\mu,\mu_0,\varphi,\varphi_0)>0$$

$$J(\mu,\varphi)=0 \text{ при } \gamma_s(\mu,\mu_0,\varphi,\varphi_0)<0. \tag{11}$$

We will consider the average values of RF of each atmospheric pixel as a solution of the task (1)–(11)

$$R_j=\frac{1}{|U_j|}\frac{\pi}{\mu_0 I_0}\iint\limits_{U_j} dx\,dy\,I(x,y,0,\tilde{\mu},\tilde{\varphi}). \tag{12}$$

The coordinates $(\tilde{\mu},\tilde{\varphi})$ define the direction of reflected SR. Then only the zenith reflection $\Omega_{\text{zenith}}(\tilde{\mu}=-1,\tilde{\varphi}=0)$ is considered, Fig. 1. It should be noted that further speculations are also correct for other directions $(\tilde{\mu},\tilde{\varphi})$.

Let us also determine the transmittance factor of the atmosphere for each Earth pixel

$$T_j=\frac{1}{|U_j|\pi I_0}\iint\limits_{U_j} dx\,dy\int_0^1 d\mu'\mu'$$
$$\int_o^{2\pi} d\varphi'\,I(x,y,H,\mu',\varphi'). \tag{13}$$

3. UNDERLYING SURFACE ALBEDO DETERMINATION

Let us find an explicit dependence of RF and US albedos. For this purpose, let us introduce basic functions for solving the following problems.

The tasks with black US:

$$\hat{T}^{3D}I^b=0, I^b(x,y,H,\mu,\varphi)=0 \text{ at } \mu<0,$$

$$I^b(x,y,0,\mu,\varphi)=$$
$$=I_0\,\delta(\mu-\mu_0)\delta(\varphi-\varphi_0) \text{ at } \mu>0. \tag{14}$$

The tasks with the US containing one white pixel and remaining black pixels:

$$\hat{T}^{3D}I_i^w=0,$$

$$I_i^w(x,y,0,\mu,\varphi)=$$
$$=I_0\,\delta(\mu-\mu_0)\delta(\varphi-\varphi_0) \text{ at } \mu>0, \tag{15}$$

$$I_i^w(x,y,H,\mu,\varphi)=$$
$$=a_i(x,y)\frac{1}{\pi}\int_0^1 d\mu'\,\mu'\int_0^{2\pi} d\varphi'\,I_i^w(x,y,H,\mu',\varphi') \text{ at } \mu<0,$$

$$a_i(x,y)=0 \text{ at } (x,y)\notin \mathrm{U}_i,\ a_i(x,y)=1 \text{ at } (x,y)\in \mathrm{U}_i.$$

The tasks with the US containing one red pixel and remaining black pixels

$$\hat{T}^{3D}I_i^r=0,\ I_i^r(x,y,0,\mu,\varphi)=0 \text{ at } \mu>0,$$

$$I_i^r(x,y,H,\mu,\varphi)=a_i(x,y)I_0 \text{ at } \mu<0. \tag{16}$$

In the tasks (14) and (15), boundary conditions (7)–(10) are used at side borders and in the task (16), zero boundary conditions are used.

Let us represent the solution of the problem (1)–(11) as a linear combination of the “black” and the “red” basic functions, the solutions of the problems (14) and (16):

$$I(x,y,z,\mu,\varphi)=$$
$$=I^b(x,y,z,\mu,\varphi)+\sum_{i=1}^{N}\gamma_i I_i^r(x,y,z,\mu,\varphi). \tag{17}$$

Really, with any values of γ_i, the function (17) complies with the equation (1) and boundary conditions (4), (7)–(10). The boundary condition (5) in the *j*-th pixel for the function (17) may be written as

$$\gamma_j=\left[\sum_{i=1}^{N}\gamma_i T_{j,i}^r+T_j^b\right]A_j,\ j=1,\dots,N, \tag{18}$$

where $T_{j,i}^r$ and T_j^b are transmittance factors (13) for the basic tasks averaged over the *j*-th Earth pixel

$$T^r_{j,i}=\frac{1}{|U_j|\pi I_0}\iint\limits_{U_j}dxdy\int\limits_0^1 d\mu'\mu'\int\limits_0^{2\pi} I^r_i(x,y,H,\mu',\varphi'),\quad(19)$$

$$T^b_j=\frac{1}{|U_j|\pi I_0}\iint\limits_{U_j}dxdy\int\limits_0^1 d\mu'\mu'\int\limits_o^{2\pi} d\varphi' I^b(x,y,H,\mu',\varphi').$$

Let us introduce two vectors and two matrices:

$$\gamma\{\gamma_j\},\ \hat{\mathbf{A}}\{A_{i,j}=A_j\,\delta_{i,j}\},$$

$$\mathbf{t}^b\{T^b_j\},\ \hat{\mathbf{T}}^r\{T^r_{j,i}\}.\quad(20)$$

Here, $\delta_{i,j}$ is the Kronecker delta. Then the system (18) takes on the following form

$$\gamma=\hat{\mathbf{A}}(\mathbf{t}^b+\hat{\mathbf{T}}^r\gamma).$$

From here, we find the vector γ of unknown values from the factorisation (17)

$$\gamma=\left(\hat{\mathbf{E}}-\hat{\mathbf{A}}\,\hat{\mathbf{T}}^r\right)^{-1}\hat{\mathbf{A}}\mathbf{t}^b.\quad(21)$$

Here, $\hat{\mathbf{E}}$ is the identity matrix. From the expression (17) let us find the RF in the j-th pixel

$$R_j=R^b_j+\sum_{i=1}^{N}\gamma_i\,R^r_{j,i}.\quad(22)$$

Here, R^b_j and $R^r_{j,i}$ are the values of RF (12) in the j-th pixel for the basic tasks

$$R^b_j=\frac{1}{|U_j|}\frac{\pi}{\mu_0 I_0}\iint\limits_{U_j}dx\,dy\,I^b(x,y,0,-1,0),$$

$$R^r_{j,i}=\frac{1}{|U_j|}\frac{\pi}{\mu_0 I_0}\iint\limits_{U_j}dx\,dy\,I^r_i(x,y,0,-1,0).\quad(23)$$

Let us rewrite the expression (22) in the matrix form

$$\mathbf{r}=\mathbf{r}^b+\hat{\mathbf{R}}^r\gamma,\quad(24)$$

where

$$\mathbf{r}\{R_j\},\ \mathbf{r}^b\{R^b_j\},\ \hat{\mathbf{R}}^r\{R^r_{j,i}\}.\quad(25)$$

By substituting (21) in (24), we obtain the relation

$$\mathbf{r}=\mathbf{r}^b+\hat{\mathbf{R}}^r\left(\hat{\mathbf{E}}-\hat{\mathbf{A}}\,\hat{\mathbf{T}}^r\right)^{-1}\hat{\mathbf{A}}\mathbf{t}^b.$$

This expression is transformed as

$$(\hat{\mathbf{E}}-\hat{\mathbf{A}}\,\hat{\mathbf{T}}^r)(\hat{\mathbf{R}}^r)^{-1}(\mathbf{r}-\mathbf{r}^b)=\hat{\mathbf{A}}\mathbf{t}^b,$$

or

$$(\hat{\mathbf{R}}^r)^{-1}(\mathbf{r}-\mathbf{r}^b)=\hat{\mathbf{A}}\left[\mathbf{t}^b+\hat{\mathbf{T}}^r(\hat{\mathbf{R}}^r)^{-1}(\mathbf{r}-\mathbf{r}^b)\right].$$

Let us introduce the vectors

$$\mathbf{u}=(\hat{\mathbf{R}}^r)^{-1}(\mathbf{r}-\mathbf{r}^b),$$

$$\mathbf{w}=\mathbf{t}^b+\hat{\mathbf{T}}^r(\hat{\mathbf{R}}^r)^{-1}(\mathbf{r}-\mathbf{r}^b).\quad(26)$$

Then the surface albedo in the j-th pixel is determined by relation of the elements of these vectors

$$A_j=u_j/w_j.\quad(27)$$

In [6], the values of albedo are obtained by representation of solution of the problem (1)–(11) as a linear combination of the “black” and “white” basic functions: solutions of the problems (14) and (15):

$$A_j=q_j/v_j,\quad(28)$$

where

$$\mathbf{q}=\hat{\mathbf{T}}^{ww}(\hat{\mathbf{R}}^{wb})^{-1}(\mathbf{r}-\mathbf{r}^b),$$

$$\mathbf{v}=\hat{\mathbf{T}}^{wb}(\hat{\mathbf{R}}^{wb})^{-1}(\mathbf{r}-\mathbf{r}^b)+\mathbf{t}^b,\quad(29)$$

$$\hat{\mathbf{R}}^{wb}\{R^{wb}_{j,i}=R^w_{j,i}-R^b_j\},\ \hat{\mathbf{T}}^{wb}\{\mathrm{T}^{wb}_{j,i}=T^w_{j,i}-T^b_j\},$$

$$\hat{\mathbf{T}}^{ww}\{T^w_{i,j}=T^w_{i,i}\,\delta_{i,j}\}.\quad(30)$$

It should be stressed that, in formulae (27), (28), the matrices $\hat{\mathbf{R}}^r$, $\hat{\mathbf{R}}^{wb}$, $\hat{\mathbf{T}}^r$, $\hat{\mathbf{T}}^{wb}$, $\hat{\mathbf{T}}^{ww}$ and the vectors $\mathbf{t}^b$, $\mathbf{r}^b$ describe reflectance and transmittance of the atmosphere for the basic tasks (14)–(16) with consideration of multiple scattering of radiation in the atmosphere and do not depend on the US albedo.

The relations (27), (26) and (28), (29) are the two variants of the main atmospheric correction formula allowing us to find the US albedo based on the values of reflected RF. Each variant of discretisation of the direct problem corresponds to each variant of the main formula. It is not necessary to find the solution of the system of non-linear equations as in the algorithm [6] and to use an iteration process.

It should be noted that, to transfer to the IPA, it is necessary to find mean values of all elements of the matrixs $\hat{\mathbf{R}}^r$, $\hat{\mathbf{T}}^r$ and the vectors $\mathbf{r}^b$ and $\mathbf{t}^b$:

$$\tilde{R}^r=\frac{1}{N}\sum_{j=1}^{N}\sum_{i=1}^{N}R^r_{i,j},\ \tilde{T}^r=\frac{1}{N}\sum_{j=1}^{N}\sum_{i=1}^{N}T^r_{i,j},$$

Table 1. Errors (%) in Determining Albedos by Formulas (27) and (28) According to Pixel Dimension d and Aerosol Optical Thickness τ^{aer} Separately for Pixels Located on and outside of the Boundaries of the Media

	τ^{aer}=0.2				τ^{aer}=0.4				τ^{aer}=0.8			
	On the media boundaries		Outside media boundaries		On the media boundaries		Outside media boundaries		On the media boundaries		Outside media boundaries	
d, km	(27)	(28)	(27)	(28)	(27)	(28)	(27)	(28)	(27)	(28)	(27)	(28)
0.25	0.031	0.027	0.091	0.077	0.075	0.069	0.071	0.073	0.071	0.065	0.41	0.26
0.5	0.059	0.053	0.039	0.04	0.12	0.11	0.07	0.07	0.27	0.21	0.1	0.1
1	0.085	0.075	0.13	0.11	0.04	0.05	0.14	0.19	1.01	0.91	1.27	1.07

$$\tilde{r}^b = \frac{1}{N}\sum_{j=1}^{N} R_j^b \,,\; \tilde{t}^b = \frac{1}{N}\sum_{j=1}^{N} T_j^b \,,$$

where $\tilde{R}^r$ and $\tilde{r}^b$ are the mean RF of an image in the tasks with red and black surfaces, $\tilde{T}^r$ and $\tilde{t}^b$ are the mean transmittance factors of an image in these problems. In IPA approximations, the formula (27) of albedo is written as

$$A_j = [R_j - \tilde{r}^b] \big/ [\tilde{R}^r \tilde{t}^b + \tilde{T}^r (R_j - \tilde{r}^b)]. \qquad (31)$$

A question whether the formulas (27) and (28) are equivalent arises. Indeed, the radiation emitted by the red pixel does not depend on the light incident on that pixel whereas the light emitted by the white pixel does. The surface of the red pixel is assumed spatially uniform whereas the surface of the white pixel is not. Below the question of equivalency of the formulas (27) and (28) is studied quantitatively,

4. QUANTITATIVE RESULTS

Let us consider the layer of standard atmosphere [8]. We will use the microphysical aerosol model developed for Belarus [9]. We will consider the extinction factor k, the scattering factor k_s and the scattering indicatrix $P(\gamma_s)$ as independent of spatial coordinates. Let us find these magnitudes using the Mi-theory [10] for wavelength λ = 0.55μm. It should be noted that the correction formulae (27) and (28) are obtained without any assumptions regarding SR wavelength.

At the first stage, the atmosphere is assumed to be transparent: for aerosol optical thickness τ^{aer} the values of 0.2, 0.4 and 0.8 are used. Rayleigh scattering optical thickness is found using the formula

$$\tau = 0.008569\lambda^{-4}(1 + 0.0113\lambda^{-2} + \\ + 0.00013\lambda^{-4})\big|_{\lambda=0.55\,\text{mkm}} \approx 0.097 \text{ [11]}.$$

Zenith angle of the Sun $\theta_0 = 40^{\circ}$ is selected.

The thickness of the atmospheric layer is assumed to be equal to 100 km. Let us consider the region [–5, 5]×[–5, 5] on the earth's surface. Three variants of dimensions of the pixel d×d are selected: d = 1 km, d = 0.5 km, and d = 0.25 km (the latter dimension complies with parameters of MODIS device). A ploughed field (A = 0.06858 [12]) with a rapeseed area located in the centre of it (A=0.153 [12]) which has a shape of a circle with radius ρ = 2 km is taken as an US.

Let us consider the case when square pixels uniformly cover the surface area. Let us introduce a mesh with step of 3 km along the height z and 0.25 km along x, y. We find the solutions of the tasks (1)–(11) and (14)–(16) by means of mesh-based discrete-ordinates method [13] in three-dimensional (x, y, z) geometry. Using formulas (19) and (23) we find reflectance and transmittance factors and form matrices (20), (25), and (30) using their values. Inverse matrices $(\hat{\mathbf{R}}^{wb})^{-1}$ and $(\hat{\mathbf{R}}^{r})^{-1}$ are found by means of Krylov subspace method. Finally, using the explicit formulas (27) and (28) we find albedos in each pixel.

Table 1 contains errors of albedo determination using formulas (27) and (28) depending on pixel dimension d and aerosol optical thickness $\tau^{aer} < 1$. The errors are listed separately for pixels located at and beyond the interface (border of the circle). The error is determined as the highest pixel variations (%) of the calculated albedos from exact ones. It is seen that accuracy of both formulae (27) and (28) are high.

With that, the errors of both formulas are comparable with the same pixel dimensions. Maximum variation of the calculated albedos from each other for all pixels increases with reduction of pixel dimension d and increase in optical thickness τ^{aer} but does not exceed 0.15 % for transparent atmosphere, Table 2.

Table 2. Deviations (%) in Calculating Albedos by Formulas (27) and (28) According to Pixel Dimension *d* and Aerosol Optical Thickness τ^{aer}

d, km	τ^{aer}=0.2	τ^{aer}=0.4	τ^{aer}=0.8
0.25	0.017	0.05	0.15
0.5	0.01	0.022	0.07
1	0.006	0.014	0.025

Then let us consider a denser atmosphere with aerosol optical thickness $1 \le \tau^{aer} \le 10$ for the case of the coarsest spatial resolution (pixel dimension *d* = 1km). With increasing τ^{aer} the influence of the surface albedo on RF becomes less, Fig. 2.

That is why the rows of the matrices $\hat{\mathbf{R}}^{wb}$ and $\hat{\mathbf{R}}^{r}$, see formulas (25) and (30), are becoming less recognisable. As consequence, the norms of the matrices used for calculation of albedo using formulas (27) and (28) are increasing, Fig. 3. That is why even low errors in solving the tasks (1)–(11) and (14)–(16) by means of mesh-based method lead to high errors of the determination of albedo. Therefore, the errors of the determination of albedo by means of both formulas (27) and (28) increase with rise of τ^{aer}, Fig. 4. Similarly higher errors of the determination of albedo will be caused even by low errors of RF measurement when processing actual measurements for higher τ^{aer}.

It should be noted that, with higher τ^{aer} for separate pixels, the error of the determination of albedo is significantly lower than maximum pixel error, compare Fig. 4 with Fig. 5. In its turn, the error of the determination of albedo in IPA approximation is much higher than the errors of formulas (27) and (28); the IPA approximation actually can be used with $\tau^{aer} < 1$.

In IPA approximation, the influence of the values of RF in neighbouring atmospheric pixels on the value of albedo in the current Earth pixel is not taken into account. To estimate this influence, let us write (27) and (26) as:

$$A_j = \left[\sum_{k=1, k\neq j}^{N} f_{j,k}(R_k - R_k^b) + f_{j,j}(R_j - R_j^b) \right] \Bigg/ \left[T_j^b + \sum_{k=1, k\neq j}^{N} g_{j,k}(R_k - R_k^b) + g_{j,j}(R_j - R_j^b) \right] \quad (32)$$

Here, $f_{j,k}$ and $g_{j,k}$ are (*j*, *k*)-th elements of the matrices $(\hat{\mathbf{R}}^r)^{-1}$ and $\hat{\mathbf{T}}^r(\hat{\mathbf{R}}^r)^{-1}$. Let us select the number *j* corresponding to central (located in the vicinity of the origin of coordinates) atmospheric and Earth pixels. Let us assign its (*x*, *y*) coordinates to each pixel number *k*. We will obtain the influence functions *f*(*x*, *y*) and *g*(*x*, *y*). Since it turns out that the absolute value of *g*(*x*, *y*) is on average less than the value of *f*(*x*, *y*) by two orders of magnitude, major contribution from the neighbouring pixels depends on the function *f*(*x*, *y*). Let us find the normalised influence function

$$\tilde{f}(x,y) = f(x,y)/\max(f(x,y)).$$

In Fig. 6 it can be seen that with increasing τ^{aer} the influence of the neighbouring pixels grows significantly.

At last, let us mention execution time. Solution of one basic task (14), (15) or (16) using a 3.2 GHz processor speed takes about $t_0 = 28$ seconds. To-

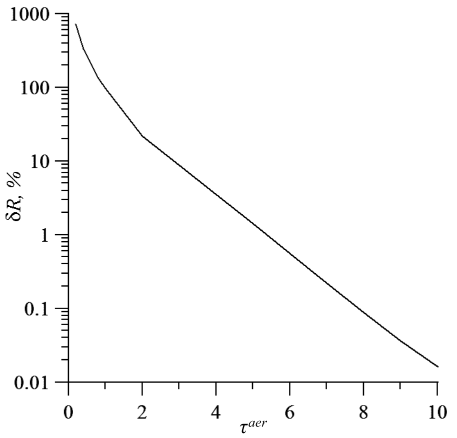

Fig. 2. Maximum pixel variations of luminance factors in problems with reflective (rapeseed area in the ploughed field) and black surfaces, *d* = 1km

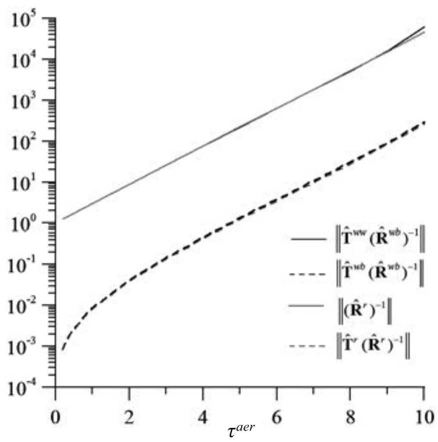

Fig. 3. Norms of the matrices used for determinating albedo, *d* = 1km

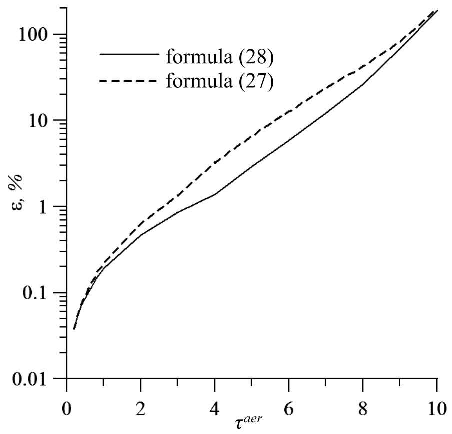

Fig. 4. Maximum pixel errors (%) of the albedo determination, d = 1km

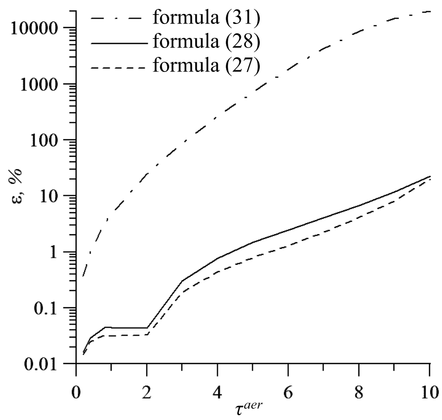

Fig. 5. Errors (%) of the determination of albedo in the central pixel, d = 1km

tal execution time of all basic problems equals to $t_0(2N+1)$ where N is the number of pixels. Time required for inversion of one matrix $\hat{\mathbf{R}}^{wb}$ or $\hat{\mathbf{R}}^{r}$ using the Krylov subspace method is about $(N/100)^4$ seconds.

5. CONCLUSION

The article presents a new variant of the main atmospheric correction formula allowing us to find the values of albedos of spatially non-uniform Lambertian surface using known values of radiance factor of the radiation reflected by the system of atmosphere and underlying surface and known atmosphere parameters. The formula is explicit and allows promptly to recover albedo. Viewing direction is contained in the formula as a free parameter.

In the new variant of the main formula, the solutions of the basic tasks with a black (non-reflective) surface and a surface containing one red pixel (isotropically emitting) and remaining black pixels are used. In the old variant, the basic problems with a underlying surface containing one white pixel (isotropically reflecting) and remaining black pixels are used. The isotropically emitting pixel is assumed spatially uniformed here and the isotropically reflecting one is not.

Two variants of discretisation of the boundary value problem for the solar light atmosphere transfer equation are proposed; each of them leads to the only solution of the problem of recovery of the Earth surface albedo.

Two variants of the formula are compared using the example of the problem of recovery of albedo of a ploughed field which includes an area with vegetation based on nadir measurements. It is demonstrated that both variants of the formula have high accuracy if the surface albedo sufficiently affects the reflected radiance factor, and their accuracy is higher than that of the variant of the formula obtained in the IPA approximation. Therefore, both variants of the formula may be used for atmospheric correction of satellite images.

When applying the presented variants of the atmospheric correction formula to actual satellite images, it is necessary to take into account that the shape of pixels is not set in this formula in any way and different pixels may have different dimensions. Therefore, after selecting a target pixel in an image (where albedo needs to be recovered), it is neces-

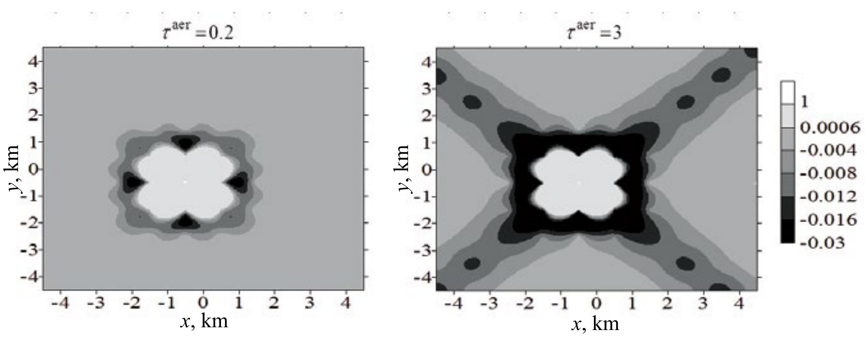

Fig. 6. Normalised influence functions $\tilde{f}(x,y)$, d = 1km

sary to set the surrounding pixels, to calculate radiance factors in them by means of any method of interpolation and then to use the main atmospheric correction formula (such methodology is specified in [7] for the "black-and-white" variant of the formula).

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