Light & Engineering
Vol. 28, No. 2, pp. 61–66, 2020

https://doi.org/10.33383/2019-034
Svetotekhnika #6, 2019, pp. 14–18

CHANGES IN IRRADIANCE AND ILLUMINANCE ON EARTH SURFACE DURING 11-YEAR SOLAR ACTIVITY CYCLE

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ABSTRACT

The article formulates the analytic expression approximating the sequence of Gregorian 11-year solar cycles and the expression of solar activity within one cycle. The dependences of effective thermodynamic temperature of the Sun photosphere and the solar constant, and the solar illuminance constant at the upper border of Earth atmosphere on the year number within one solar cycle were obtained. The generalised analytic expression for integral transmittance of atmosphere (within its spectral window) on the Earth surface for the direct and diffuse components of solar radiation and their sums at different solar altitude angles is presented. The analytic expressions of dependences of irradiance and illuminance on the Earth surface within spectral window of atmosphere and within the visible region of solar radiation spectrum on the year number within a certain solar cycle at different solar altitude angles are obtained. The results of calculation of direct and diffuse components of irradiance and illuminance and their sums in the case of clear sky are presented for example. The proposed approach allows similar calculations to conduct for different types of sky cover.

Keywords: 11-year solar cycle, solar constant, solar illuminance constant, solar altitude angle, cyclic changes, direct and diffuse components of irradiance and illuminance, Earth surface

The 11-year solar activity cycle significantly effects energy and light and engineering characteristics of solar radiation (SR) on the Earth surface and defines the nature of all aspects of human activities to a large extent.

The changes of SR in the visible region of spectrum in the spectral window of Earth atmosphere (SWAT) affect daily activity of human neuroendocrine system defining the diurnal rhythms of all biological systems of body. Changes of diurnal rhythms cause changes of daily intellectual activity of human including, in particular, the process of visual perception and building of sensing model of surrounding objects on the basis of it as well as concrete and abstract thinking.

By now, the 11-year cyclic changes in irradiance and illuminance on the Earth surface reaching 30 % and significantly affecting visual perception processes have not been considered in light engineering practice.

This work aims at obtaining analytic expressions describing 11-year cyclic changes in a number of energy (radiometry) and light engineering (photometry) characteristics in different spectral regions of SR at different solar altitude angle[1].

Solar activity is measured by solar activity index characterised by the Wolf number $W = k(10g + s)$, where s is the number of individual spots on the

[1] It was assumed that the obtained results might be:

– Used in studies and forecasts of conscious and unconscious reactions of human body to cyclic changes of solar radiation characteristics including those related to operation of human vision system;

– Taken into account in works for standardization of natural indoor lighting not only for comfortability of visual performance but also for prevention and/or elimination of deviations in human body circadian systems (necessity of such works is demonstrated in the recent review [1]).

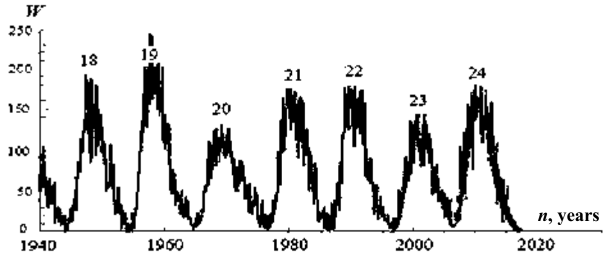

Fig. 1. Wolf number W dependence on the time in Gregorian calendar (numbers 18–24 mark solar cycles from 1940 to 2019 [3–5])

Sun photosphere surface, g is the number of sunspot groups and k is the factor usually taken equal to one [2, 3].

W and its dependence on time characterise the main and the most pronounced 11-year solar cycle, the Schwabe cycle (Fig. 1). Significantly less pronounced cycles (in particular, the 22-year Hale cycle) will not be taken into account.

During minimum solar activity periods, W (= W_{min}) is virtually constant and equals to 0–15. On the contrary, maximum solar activity is characterised by variability of W (= W_{max}) equal to 120–250. For the purposes of this article, the values of W_{min} = 10 and W_{max} = 180 averaged over the period from 1940 to 2019 were taken. Moreover, solar activity wave front and fall times were taken equal to each other, which does not cause significant error in the obtained results and allows us to approximate the dependence of W on year number N (with taken assumptions) by means of a sinusoidal function:

$$W(N) = W_{\text{aver}}\left\{1 + 0,895 \cdot \sin\left[\frac{2\pi(N-1755)}{11} - \frac{\pi}{2}\right]\right\}, \quad (1)$$

where W_{aver} =0.5(W_{min} + W_{max}), 1755 is the year of minimum solar activity (beginning of the zero cycle) and close to the year of start of regular studying of cyclic changes of solar activity (~1749).

Dependence of W on year number n within a certain 11-year cycle of solar activity has the following form:

$$W(n) = W_{\text{aver}}\left[1 + 0,895 \cdot \sin\left(\frac{2\pi n}{11} - \frac{\pi}{2}\right)\right]. \quad (2)$$

The $W(n)$ dependency graph based on the expression (2) is presented in Fig. 2.

Cyclic changes in W lead to cyclic change of the Sun photosphere radiant luminocity M_{eS} [$T_{eff}(n)$], where T_{eff} is the equilibrium effective thermodynamic temperature of the Sun photosphere radiation.

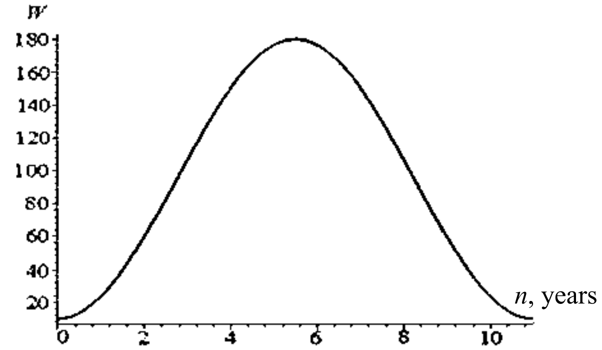

Fig. 2. Changes in Wolf number W within a certain 11-year solar cycle

The Planck's radiation model used in the most cases as the model of the Sun photosphere radiation [6, 7] with spectral radiance of m_{eS} (λ, T_{eff}) is expressed as

$$m_{eS}(\lambda, T_{eff}) = C_1\lambda^{-5}\left(\exp\frac{C_2}{\lambda T_{eff}} - 1\right)^{-1}, \quad (3)$$

where $C_1 \approx 3.742 \cdot 10^{-16}$ Wm^2 and $C_2 \approx 1.439 \cdot 10^{-2}$ mK.

At that:

– Availability of absorption spectral lines in the Sun photosphere radiation spectrum (Fraunhofer lines) [8] and in the Earth atmosphere is significant only for spectroscopy but do not significantly affect results of light engineering calculations;

– It is obvious that T_{eff} is a function of n in (3). Nevertheless, in accordance with the recommendations of the International Radiation commission [9], in (3) and in the integral of m_{eS} (λ, T_{eff}) over λ in the form of M_{eS} (T_{eff}), luminosity of dependence $T_{eff}(n)$ is not taken into account and the value of T_{eff} is taken equal to a certain constant, which does not allow us to define dependences of energy and light-engineering characteristics of SR on the Earth surface (ES) on n.

As a basis for determination of these dependencies on the Earth surface, the value of solar constant $E_{e.SC}$ (T_{eff}) in the form

$$E_{e.\text{SC}}(T_{eff}) = \int_0^\infty e_{eS}(\lambda, T_{eff})\,d\lambda = \\ = \left(\frac{r}{R}\right)^2 \int_0^\infty m_{eS}(\lambda, T_{eff})\,d\lambda, \quad (4)$$

was used, which is the irradiance of an area located on the upper border of Earth atmosphere with nor-

Table 1. Values of Solar Constant in Different Spectral Regions and Solar Illuminance Constant at Minimum and Maximum Solar Activity

Nature of solar activity	$E_{e.SC}$, Wm^{-2}			$E_{v.SC}$, lx
	at $0 \le \lambda \le \infty$ nm	at $300 \le \lambda \le 1200$ nm	at $350 \le \lambda \le 770$ nm	at $350 \le \lambda \le 770$ nm
Minimum	1106.3	838.7	514.0	135110
Maximum	1369.2	1057.4	665.8	173600
Average	1237.7	948.1	589.9	154355

mal incidence of SR. Here, e_{eS} (λ, T_{eff}) is the spectral irradiance on the border of Earth atmosphere, $r = 6.96 \cdot 10^5$ km is the equatorial radius of the Sun, $R = 1.496 \times 10^{12}$ km is the radius of the Earth circular orbit [7].

According to the data of satellite actinometric measurements referring to maximum values of solar cycles 20 and 21, the most probable value of $E_{e \cdot SC}$ (T_{eff}) is 1368–1377 Wm^{-2} provided there are no regular temporal changes, which allows us to use the term “solar constant”. The value $E_{e \cdot SC, max}(T_{eff}) \approx 1370$ Wm^{-2} taken from the 1956 International Pyrheliometric Scale [7, 9] is taken as a standard value of this indicator. According to (3) and (4), this value corresponds with the value $T_{eff, max} = 5780$ K.

Solar illuminance constant $E_{v \cdot SC}$ may be expressed in accordance with (4) as

$$E_{v.SC}\left(T_{eff}\right) = 683 \int_{350}^{770} e_{eS}(\lambda, T_{eff}) V\left(\lambda\right) d\lambda = \left(\frac{r}{R}\right)^2 683 \int_{350}^{770} m_{eS}(\lambda, T_{eff}) V\left(\lambda\right) d\lambda, \quad (5)$$

and it is the irradiance of an area located on the upper border of Earth atmosphere with normal incidence of SR. In conditions of minimum solar activity, $E_{v.SC} = E_{v.SC, min} = 135110$ lx [10, 11], which corresponds with values of $E_{e \cdot SC, min}$ of 1106 Wm^{-2} and $T_{eff, min}$ of 5480 K. In accordance with (3) and (4), the previously defined value $E_{SC, max} \approx 1370$ Wm^{-2} with $T_{eff, max} = 5780$ K corresponds to the value $E_{v.SC, max} = 173600$ lx.

With known $T_{eff, min}$ and $T_{eff, max}$, the dependence $T_{eff}(n)$ within a certain 11-year solar cycle is expressed as

$$T_{eff}\left(n\right) = T_{eff, aver}\left[1 + 0,027 \cdot \sin\left(\frac{2\pi n}{11} - \frac{\pi}{2}\right)\right], \quad (6)$$

where $T_{eff, aver} = 0.5(T_{eff, min} + T_{eff, max})$. The chart of this dependence is shown in Fig. 3 and $E_{e.SC, min}$ and $E_{e.SC, max}$ in different spectral regions are shown in Table 1.

According to (4), with calculated values of $E_{e.SC, min}$ and $E_{e.SC, max}$ for spectral regions (0–∞) nm, (300–1200) nm (SWAT [7, 12]) and the visible spectrum (350–770) nm in SWAT, dependencies $E_{e.SC}(n)$ have the following form

$$E_{e.SC}\left(n\right) = E_{e.SC, aver}\left[1 + 0,1062 \cdot \sin\left(\frac{2\pi n}{11} - \frac{\pi}{2}\right)\right]$$

$$\text{at } 0 \text{ nm} \le \lambda \le \infty \text{ nm}, \quad (7)$$

$$E_{e.SC}\left(n\right) = E_{e.SC, aver}\left[1 + 0,1153 \cdot \sin\left(\frac{2\pi n}{11} - \frac{\pi}{2}\right)\right]$$

$$\text{at } 300 \text{ nm} \le \lambda \le 1200 \text{ nm}, \quad (8)$$

$$E_{e.SC}\left(n\right) = E_{e.SC, aver}\left[1 + 0,1287 \cdot \sin\left(\frac{2\pi n}{11} - \frac{\pi}{2}\right)\right]$$

$$\text{at } 350 \text{ nm} \le \lambda \le 770 \text{ nm}, \quad (9)$$

where $E_{e.SC, aver} = 0,5(E_{e.SC, min} + E_{e.SC, max})$. The values of $E_{e.SC, aver}$ for each spectral region are shown in the last line of Table 1 and the graphs of dependencies (7)–(9) are shown in Fig. 4.

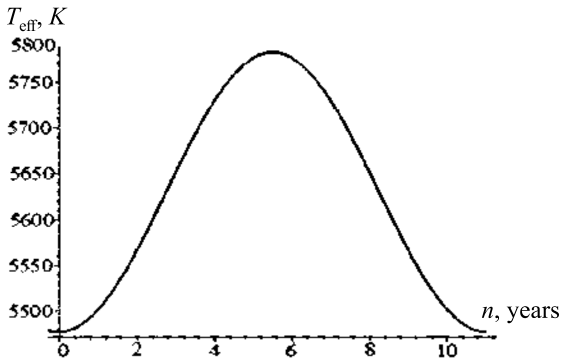

Fig. 3. Changes in effective thermodynamic temperature of Sun photosphere radiation T_{eff} within a certain 11-year solar cycle

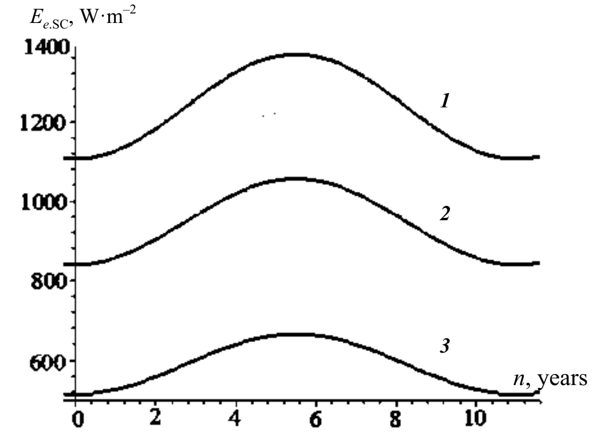

Fig. 4. Changes in solar constant $E_{e.SC}$ in different spectral regions of solar radiation within a certain 11-year solar cycle: (0–∞) nm (*1*), (300–1200) nm (*2*), (350–770) nm (*3*)

According to (5), the dependence $E_{v\cdot SC}(n)$ within a certain 11-year solar cycle has the following form

$$E_{v.SC}(n) = E_{v.SC.aver}\left[1 + 0{,}1247\cdot\sin\left(\frac{2\pi n}{11} - \frac{\pi}{2}\right)\right], \quad (10)$$

where the value of $E_{v\cdot SC,\ aver}$ is given in the last column of Table 1 and the graph is shown in Fig. 5.

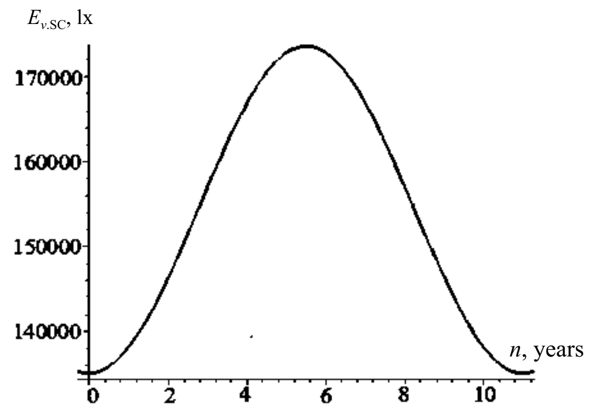

Fig. 5. Changes in solar illuminance constant $E_{e.SC}$ in visible spectrum within a certain 11-year solar cycle

Since SR is distributed in SWAT of (300–1200) nm [7, 12], for determination of irradiance, $E_{e.ES}(n)$, and illuminance, $E_{v.ES}(n)$, on the Earth surface in the visible spectrum of (350–770) nm requires only to take integral transmittance of atmosphere into account.

Since direct solar radiation generates the direct and the diffuse components of SR, let us consider two integral coefficients of atmosphere transmittance corresponding to them, τ_{dir} and τ_{diff}, which depend on solar altitude angle h in a calculated point of Earth surface. Availability of dependencies $\tau_{dir}(h)$ and $\tau_{diff}(h)$ causes availability of corresponding dependencies $T_{eff,\ dir}(h)$ и $T_{eff,\ diff}(h)$, $e_{eS,\ dir}[\lambda, n, T_{eff}(h)]$, $e_{eS,\ diff}[\lambda, n, T_{eff}(h)]$, $E_{e.ES,\ dir}(n, h)$, $E_{e.ES,\ diff}(n, h)$, $E_{v.ES,\ dir}(n, h)$ and $E_{v.ES,\ diff}(n, h)$.

The analysis of data [13] showed that, in different conditions of sky cover (including clear sky) and the surface, the expressions for $E_{e.ES,\ dir}(n, h)$, $E_{e.ES,\ diff}(n, h)$, $E_{v.ES,\ dir}(n, h)$ и $E_{v.ES,\ diff}(n, h)$ with any n of a certain solar cycle (e.g. at n = 0 or 11 corresponding to minimum solar cycle) may be expressed in general form

$$E_{ES}(h) = a[1 + \sin(bh - c)]. \quad (11)$$

The values of E_{ES} (90°) for all types of sky cover and for clear sky are defined by approximating data [13] by the expression (11) with further extrapolation up to the value h = 90°.

It is obvious that expressions of dependencies $\tau_{dir}(h)$ and $\tau_{diff}(h)$ have general form

$$\tau(h) = 0{,}5\cdot\tau(90^\circ)\cdot[1 + \sin(bh - c)], \quad (12)$$

where τ(90°) is the integral coefficient of atmosphere transmittance for normal incidence of solar radiation on Earth surface in SWAT and visible spectrum at the equatorial latitude during the vernal and autumnal equinoxes and equal to ratios $E_{v.ES,\ min}(90^\circ)$ / $E_{v.SC,\ min}(90^\circ)$ or, respectively, $E_{e.ES,\ min}(90^\circ)$ / $E_{e.SC,\ min}(90^\circ)$.

With consideration of the expression (11), the values of $E_{e.ES,\ dir}(n, h)$, $E_{e.ES,\ diff}(n, h)$, $E_{v.ES,\ dir}(n, h)$ and $E_{v.ES,\ diff}(n, h)$ for different types of sky cover and different degree of sky cover in visible spectrum are described as

$$E_{e.ES\,i,j}(n,h) = E_{e.SC}(n)\cdot\tau_{i,j}(h), \quad (13)$$

$$E_{v.ES\,i,j}(n,h) = E_{v.SC}(n)\cdot\tau_{i,j}(h), \quad (14)$$

where i and j indices correspond to different types and degrees of sky cover respectively.

As an example, Table 2 contains the values of $E_{e.ES,\ dir}(n)$, $E_{e.ES,\ diff}(n)$, $E_{v.ES,\ dir}(n)$ and $E_{v.ES,\ diff}(n)$ without sky cover in SWAT and in visible region in conditions of minimum and maximum solar activity.

The data shown in Table 2 is obtained using the values $\tau_{dir}(90^\circ)$ = 0.729 and $\tau_{diff}(90^\circ)$ = 0.205 for clear sky after approximation and further extrapolation of data [13].

Table 2. Direct and Diffuse Irradiance and Illuminance on the Earth Surface at Minimum and Maximum Solar Activity without Sky Cover in Spectral Window of Atmosphere Transparency and in Visible Spectrum

Nature of solar activity	At $300 \le \lambda \le 1{,}200$ nm		At $350 \le \lambda \le 770$ nm		At $350 \le \lambda \le 770$ nm	
	$E_{e.ES,\ dir}$, Wm^{-2}	$E_{e.ES,\ diff}$, Wm^{-2}	$E_{e.ES,\ dir}$, Wm^{-2}	$E_{e.ES,\ diff}$, Wm^{-2}	$E_{v.ES,\ dir}$, lx	$E_{v.ES,\ diff}$, lx
Minimum	611.2	172.3	374.6	105.6	98470	27750
Maximum	770.6	217.2	485.2	136.8	126520	35660
Average	690.9	194.7	429.9	121.2	112490	31700

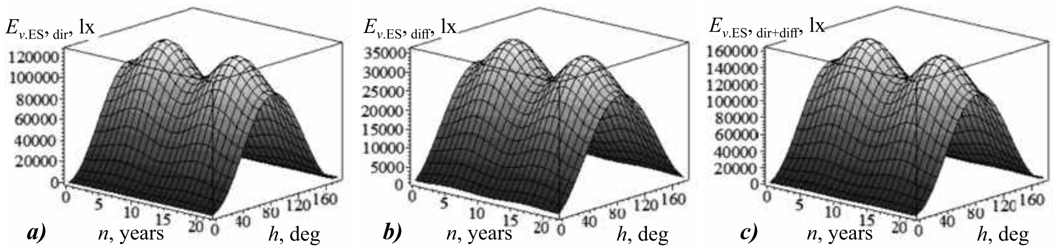

Fig. 6. Dependencies of direct (*a*) and diffuse (*b*) components of illuminance on the Earth surface and their sum (*c*) on solar angle latitude *h* and year number *n* within two 11-year solar cycles

In conditions of clear sky, in SWAT (300–1200) nm, the dependences $E_{e.ES,\ dir}(n,\ h)$, $E_{e.ES,\ diff}(n,\ h)$ and their sum $E_{e.ES,\ dir+diff}(n,\ h)$ have the form

$$E_{e.ES,\mathrm{dir}}(n,h)= =0{,}5E_{e.ES,\mathrm{aver}}\left[1+\sin(0{,}035h-1{,}473)\right]\times \times\left[1+0{,}1153\sin\left(\frac{2\pi n}{11}-\frac{\pi}{2}\right)\right], \quad (15)$$

$$E_{e.ES,\mathrm{diff}}(n,h)= =0{,}5E_{e.ES,\mathrm{aver}}\left[1+\sin(0{,}030h-1{,}094)\right]\times \times\left[1+0{,}1153\sin\left(\frac{2\pi n}{11}-\frac{\pi}{2}\right)\right], \quad (16)$$

$$E_{e.ES,\mathrm{dir+diff}}(n,h)=E_{e.ES,\mathrm{dir}}(n,h)+E_{e.ES,\mathrm{diff}}(n,h). \quad (17)$$

The values of $E_{e.ES,\ aver}$ in (15)–(17) and the following expressions (18)–(23) are shown in the last line of Table 2.

In the spectral region of (350–770) nm for clear sky, the similar dependences and their sum have form

$$E_{e.ES,\mathrm{dir}}(n,h)= =0{,}5E_{e.ES,\mathrm{aver}}\left[1+\sin(0{,}035h-1{,}473)\right]\times \times\left[1+0{,}1287\sin\left(\frac{2\pi n}{11}-\frac{\pi}{2}\right)\right], \quad (18)$$

$$E_{e.ES,\mathrm{diff}}(n,h)= =0{,}5E_{e.ES,\mathrm{aver}}\left[1+\sin(0{,}030h-1{,}094)\right]\times \times\left[1+0{,}1287\sin\left(\frac{2\pi n}{11}-\frac{\pi}{2}\right)\right], \quad (19)$$

$$E_{e.ES,\mathrm{dir+diff}}(n,h)=E_{e.ES,\mathrm{dir}}(n,h)+E_{e.ES,\mathrm{diff}}(n,h). \quad (20)$$

In the spectral region of (350–770) nm for clear sky, the dependences $E_{v.ES,\ dir}(n,\ h)$ and $E_{v.ES,\ diff}(n,\ h)$ have form

$$E_{v.ES,\mathrm{dir}}(n,h)= =0{,}5E_{v.ES,\mathrm{aver}}\left[1+\sin(0{,}035h-1{,}473)\right]\times \times\left[1+0{,}1247\sin\left(\frac{2\pi n}{11}-\frac{\pi}{2}\right)\right], \quad (21)$$

$$E_{v.ES,\mathrm{diff}}(n,h)= =0{,}5E_{v.ES,\mathrm{aver}}\left[1+\sin(0{,}030h-1{,}094)\right]\times \times\left[1+0{,}1247\sin\left(\frac{2\pi n}{11}-\frac{\pi}{2}\right)\right], \quad (22)$$

$$E_{v.ES,\mathrm{dir+diff}}(n,h)=E_{v.ES,\mathrm{dir}}(n,h)+E_{v.ES,\mathrm{diff}}(n,h). \quad (23)$$

As an example, Fig. 6 shows the graphs of dependencies $E_{v.ES}(n,\ h)$ based on expressions (21)–(23) within two cycles of solar activity.

The forms of the graphs of dependencies $E_{e.ES}(n,\ h)$ based on (15)–(20) are similar to the forms of the graphs shown in Fig. 6 but with minimum and maximum values of $E_{e.ES}$ presented in Table 2.

This approach allows us to obtain similar results for nine types of sky cover (A_c, C_b, C_c, C_i, C_s, C_u, N_s, S_c, S_t) and for four values of sky cover degree considered in [13].

The obtained results allow us to clarify changes of energy and light-engineering characteristics of SR on the Earth surface at any date of a certain year within an 11-year solar cycle as well as to increase accuracy of calculations of natural irradiance and illuminance on the Earth surface.

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