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ANALYTIC REPRESENTATION OF RELATION BETWEEN SOLAR ALTITUDE ANGLE AND LOCAL TIME FOR CALCULATING DAYLIGHT IRRADIANCE AND ILLUMINANCE OF THE EARTH SURFACE

Alexander V. Leonidov

Senior citizen, Former Researcher of VNISI named after S.I. Vavilov
E-mail: avleonidoff@mail.ru

ABSTRACT

The analytic expression which sets relation between the solar altitude angle and local time at a random point of the Earth surface on a random day of a year is obtained. The obtained expression and derived relations allow one to conduct calculations of daylight irradiance and illuminance of the Earth surface in analytic form.

Keywords: daylight irradiance and illuminance, Earth surface, geocentric equatorial system, analytic form of representation, solar altitude angle, local time, geographic coordinates, year day

1. INTRODUCTION

Daylight illuminance and irradiance of the Earth surface cause determining influence on characteristics of stationary and none-stationary visual processes and characteristics of visual performance [1, 2]. Irradiance caused by solar radiation reaching the surface of the Earth controls circadian activity of human body.

In the course of research of the listed processes, it is quite often necessary to use relation between daily values of solar altitude angle h and local time t_{local} at a design point of the Earth surface with random values of latitude φ on a random day n of a year.

The only publication that presents such data to the fullest extent in the last century was the unique publication [3], which contained the results of the studies conducted in the astrophysical laboratory of the Leningrad University under supervision of Professor V.V. Sharonov. The tabular data presented in [3] include information on relation between solar altitude angle and time of the day with discretisation interval Δt = 1 hour for different days of a random year with discretisation interval Δn = 10 days as well as for the values of latitude within the range of $35° \leq \varphi \leq 70°$ with discretisation interval $\Delta\varphi = 5°$. Tabular representation of data makes it necessary to interpolate within and extrapolate beyond the discretisation intervals used in [3] respectively in the course of lighting engineering calculations. Moreover, tabular representation of data does not allow us to perform lighting engineering calculations in analytic form, which complicates possibilities to use the obtained results and to interpret them substantially.

Nowadays, the copyright holder of [3], which is a rare book, is the University of California. That arouses doubts on the possibility of the book [3] republication in Russia.

Inaccessibility of [3] and general trend of mathematical formalisation of results of lighting engineering studies have defined the main goal of this article: analytic representation of the data on relation between solar altitude angle and local time at a random point of the Earth surface required for calculating its daylight irradiance and illuminance.

Another goal of the work related to active exploration of the polar regions of the Earth is necessity of widening the range of values of latitudes of the Earth surface between the Equator and the North and the South poles of the Earth.

2. USED DATA AND METHODS

Daily and yearly changes of characteristics of illuminance and irradiance of the Earth surface are conditioned by a large amount of spatial motions of the Earth with the most significant of them being daily rotation of the Earth and its movement around the Sun characterised by low orbit eccentricity of the Earth e = 0.0167 (the shape of the Earth orbit is nearly circular) [4]. Exclusion of less important motions (Earth axis precession and nutation, etc.) out of consideration, don't introduce any significant errors in the results of lighting engineering calculations.

To obtain an analytic expression relating solar altitude angle during daytime and geographic coordinates of a design point of the Earth surface and local time of central meridian $\xi_{central}$ of a time zone N corresponding with it, the spherical model of the Earth and geocentric equatorial system of the II type [5–7] presented in Fig. 1 were used.

This coordinate system is a projection of geographic coordinates of the Earth surface on an imaginary celestial sphere. The plane perpendicular to the main axis and crossing the centre of the Earth is the main plane, the celestial equator plane, which divides the celestial sphere into the northern and the southern celestial hemispheres and is a projection of the Earth equator on the celestial sphere. The celestial points P and $P^/$ lie in the celestial meridian plane. The axis P, $P^/$ is the main axis (celestial axis) coincident with the Earth axis. The celestial meridian is a projection of the earth meridian plane on the celestial sphere at a design point of the Earth surface.

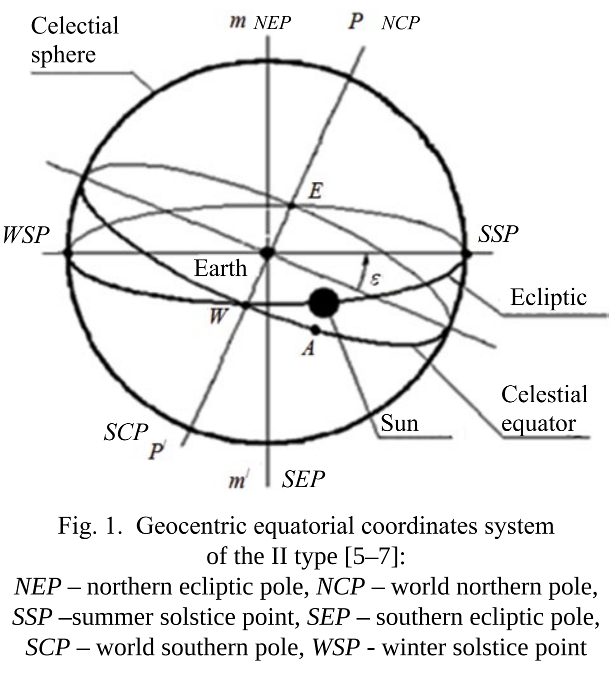

Fig. 1. Geocentric equatorial coordinates system of the II type [5–7]:
NEP – northern ecliptic pole, *NCP* – world northern pole, *SSP* –summer solstice point, *SEP* – southern ecliptic pole, *SCP* – world southern pole, *WSP* - winter solstice point

Visible yearly motion of the Sun centre occurs along the ecliptic. The points m and $m^/$ are the northern and the southern ecliptic poles (NEP and SEP respectively). The angle ε between the ecliptic plane and the celestial equator plane equals to ε = 23.45° [4]. The ecliptic and the celestial equator cross each other at two points: at the western point W which is the vernal equinoctial point and at the eastern point E which is the autumnal equinoctial point. On a random day of the year, the point corresponding to the current position of the Sun centre lies in the celestial meridian plane crossing the celestial axis P, $P^/$.

The great circle of the celestial sphere (declination circle) is the celestial meridian crossing the centre of the Sun, the celestial axis P, $P^/$ and crossing the celestial equator at point A. Angular distance between the declination circle and the vernal equinoctial point W measured along the celestial equator is the Sun's right ascension measured in time units, and the angular distance between the centre of the Sun and the point A of the celestial equator in the declination circle is the solar declination (solar declination is positive north of the celestial equator and negative south of the celestial equator).

Since lighting engineering calculations are performed on the surface of the Earth, hereinafter time is used with day duration of 24 hours [4], and the moment of the Sun inferior culmination at the vernal equinoctial point corresponding to 00 hours and 00 minutes of the local time of a selected time zone

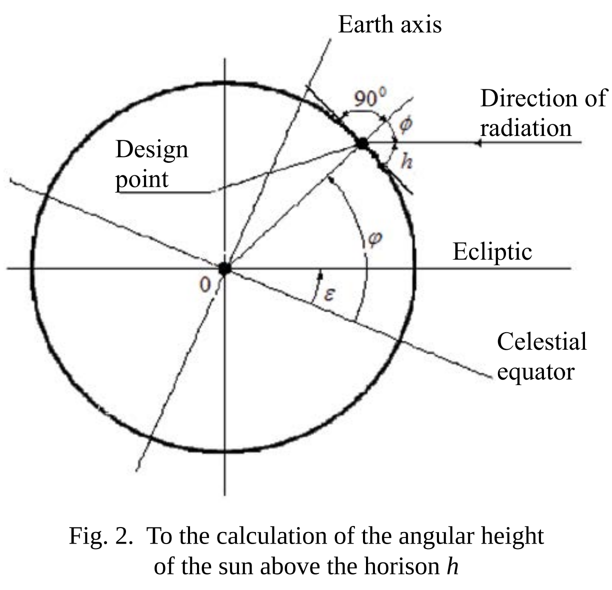

Fig. 2. To the calculation of the angular height of the sun above the horison h

is taken as zero time of the day. The moment of the Sun inferior culmination at the vernal equinoctial point is also taken as a point corresponding to the reference point of year day numbers.

It is convenient to use the projection of the solar declination circle on the Earth surface presented in Fig. 2 to determine the dependence of solar altitude angle on local time.

Solar altitude angle h relative to the horizon (without consideration of solar radiation refraction) is measured within the solar declination circle corresponding to a specific day of the year, current local time and geographic coordinates of the design point of Earth surface.

In the case of the northern hemisphere of the Earth, solar altitude angle above the horizon [5–7] equals to

$$h = 90^0 - \varphi + \varepsilon. \quad (1)$$

For the southern hemisphere, this relation is written as:

$$h = 90^0 + \varphi - \varepsilon. \quad (2)$$

In the relations (1, 2), ε is the angle between the celestial equator plane coincident with the Earth equator plane and the ecliptic plane.

3. RESULTS

Low eccentricity of the Earth orbit making Earth angular velocity relative to the Sun almost constant, the assumptions described above as well as the relations (1, 2) allow us to obtain representation of the sought dependence of solar altitude angle on local time in the geocentric equatorial system. This dependence is represented in the form of the sum of two periodic components corresponding to yearly orbital motion of the Earth around the Sun and daily rotation of the Earth. For the northern and the southern hemispheres, these expressions are written respectively as:

$$h(n,\varphi,t_{local}) = \\ = \varepsilon \sin \Phi(n) - (90^0 - \varphi)\cos\psi(t_{local}), \quad (3)$$

$$h(n,\varphi,t_{local}) = \\ = -\varepsilon \sin \Phi(n) + (90^0 + \varphi)\cos\psi(t_{local}), \quad (4)$$

where Φ is the current phase of yearly orbital motion of the Earth around the Sun, n is the number of full days since zero time, ψ is the current phase of the daily Earth rotation, t_{local} is the local time corresponding to the standard (winter) time for the central meridian ξ_{centre} of the time zone N where the design point of the Earth surface is located, φ is the latitude of the design point.

It is obvious that, in the specified conditions and with consideration of the taken zero time equal to 00 hours and 00 minutes of vernal equinox in the northern hemisphere of the Earth, the expressions for phases Φ (n) and ψ (t_{local}) in the relations (3) and (4) are written as $\Phi(n)=\dfrac{2\pi n T_{da}}{T_{year}}$, $\psi(t_{local})=\dfrac{2\pi t_{local}}{T_{day}}$.

In the relations for phases Φ (n) and ψ (t_{local}), $T_{day} = 24\ h$ is duration of the day, $T_{year} = 8760\ h$ [4] is duration of the year.

Standard (winter) local time t_{local} ($0 \le t_{local} \le 24$ hours) for the central meridian of a random time zone N ($0 \le N \le 23$) is described by means of the following relation:

$$t_{local} = UTC + N, \quad (5)$$

where UTC is the Coordinated Universal Time.

The data related to the northern hemisphere of the Earth are of the greatest interest for Russian specialists.

With consideration of the above mentioned expressions for current values of phase Φ (n) and ψ (t_{local}), the relation (3) allows one to calculate dependence of the solar altitude angle on local time for random geographic coordinates of the Earth surface in a specific time zone N.

The inverse function for the relation (3) which describes dependence of local time t_{local} (h) and solar altitude angle for values of n and φ within the ranges of $0 \le n \le 365$ and $0° \le \varphi \le 90°$ is written as:

$$t_{local}(h) = \\ = \begin{cases} \dfrac{T_{day}}{2\pi} \arccos\left[\dfrac{1}{(90-\varphi)}\left(\varepsilon \sin \dfrac{2\pi n T_{day}}{T_{year}} - h\right)\right] \text{ first} \\ \text{half of the day,} \\ \dfrac{T_{day}}{2\pi}\left\{2\pi - \arccos\left[\dfrac{1}{(90-\varphi)}\left(\varepsilon \sin \dfrac{2\pi n T_{day}}{T_{year}} - h\right)\right]\right\} \\ \text{second part of the day.} \end{cases} \quad (6)$$

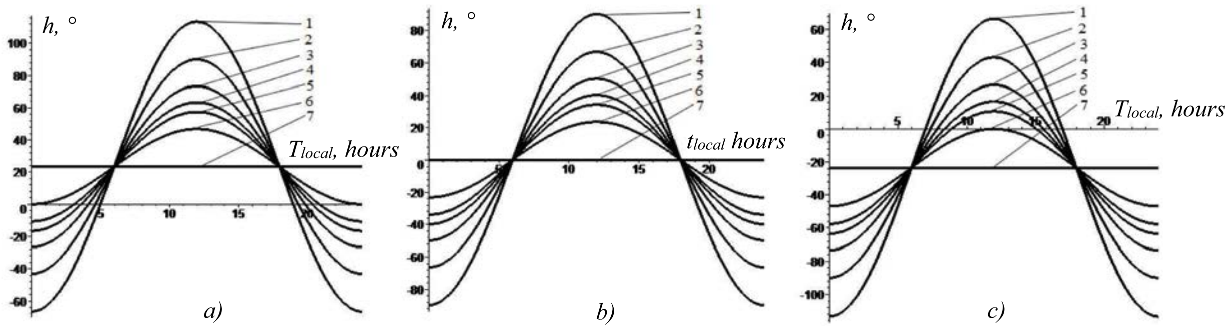

Fig. 3. Dependences of solar altitude angle on local time and geographic latitude of the design point of the Earth surface: ***a)*** n = 91 (June 22), ***b)*** n = 0 (March 21) and n = 182 (September 22), ***c)*** n = 273 (December 22), 1 – φ = 00.00°, 2 – φ = 23.45°, 3 – φ = 40.00°, 4 – φ = 50.00°, 5 – φ = 55.90°, 6 – φ = 66.55°, 7 – φ = 90.00°

The expressions for calculating local sunrise time t_{srise} and sunset time t_{sset} as well as day duration ($t_{sset} - t_{srise}$) directly follow from the relation (6) with h = 0°.

The relation (3) allows us also to calculate the values of solar altitude angle corresponding to the superior culmination (t_{local}=12 hours) and the inferior culmination (t_{local}=00 hours) of the Sun with random combinations of the values of n and φ which lie within the ranges $0 \le n \le 365$ and $0° \le \varphi \le 90°$.

The following relation describes solar altitude angle for the superior culmination:

$$h_{culm.top} = \varepsilon \sin \frac{2\pi n T_{day}}{T_{year}} + \left(90^0 - \varphi\right), \qquad (7)$$

and the following one describes that for the inferior culmination:

$$h_{culm.lower} = \varepsilon \sin \frac{2\pi n T_{day}}{T_{year}} + \left(-90^0 + \varphi\right). \qquad (8)$$

4. DISCUSSION OF RESULTS

As an example, Fig. 3 shows dependences of solar altitude angle and local time as per the expression (3) for different values of geographic latitude φ on the day of summer solstice (n = 91), on the days of vernal and autumnal equinoxes (n = 0, n = 182) and on the day of winter solstice (n = 273).

The projections of the nodes of the curve set presented in Fg. 3 on the local time axis correspond to sunrise and sunset times at a latitude φ = 23.45° on the vernal and autumnal equinox days.

The intercepts of the curves in Fig. 3 with the time axis correspond to local sunrise and sunset times for the central meridian of the considered time zone N and fully correspond to values calculated using the relation (6) with h = 0°.

On the day of the winter solstice in the northern hemisphere, increase in the values of latitude from φ = 00.00° to φ = 90.00° in the expression (7) leads to decrease in maximum daily values of solar altitude angle from h = 66.55° to h = –23.45°. On the days of the vernal equinox n = 0 (March 21) or the autumnal equinox n=182 (September 22), on the days of the summer solstice n = 91 (June 22) and the winter solstice n = 273 (December 22) in the northern hemisphere, the increase in values of latitude of the design point of the Earth surface in the expression (7) from φ = 00.00° to φ = 90.00° lead to decrease in maximum values of solar latitude angle from h = 90.00° to h = 00.00°, from h = 113.45° to h = 23.45° and from h = 66.55° to h = –23.45° respectively.

Comparison of the results of the calculations using the relations (3, 4, 6–8) shows that they are fully coincident with the data provided in [3]. The results of the calculations using the expressions (3, 4, 6–8) also fully comply with the description of instantaneous position and motion of the Sun along the imaginary celestial sphere provided in [5, 7].

Similar results may be obtained using the above mentioned relations for any values of latitude in the northern and southern hemisphere of the Earth on a random day of the year.

5. CONCLUSION

The results obtained in this work allow us to formalise and significantly simplify calculations of daylight irradiance and illuminance of the Earth surface, to make it possible to determine the analytic form of the influence of solar radiation on characteristics of different visual processes, visual performance and circadian activity of human body. Moreover, the results of the work may be used in other areas of human activity, e.g. in architecture, biophysical studies, research of different aspects of relations between the Sun and the Earth and geophysical processes, in climatology, etc.

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Alexander V. Leonidov,
Ph.D. in Technical Science, graduated from MPEI in 1970 by speciality light and engineering and sources of light. Currently, he is a retired freelance researcher