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OPTIMIZATION METHODS FOR SPECTRAL SYNTHESIZING OF A TUNEABLE COLOUR LIGHT SOURCE

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ABSTRACT

A method for a tuneable colour light source (TCLS) output spectrum synthesizing is described. A TCLS is a multi-channel LED light source, which is able to mimic and produce different spectral distributions and can be used for the realization of different spectra, e.g. the spectra of different CIE standard illuminants. The synthesizing of output spectrum is actually an optimization problem of tuning the output spectrum of a TCLS to the target spectrum. It is also a so called constrained problem as output spectrum is produced by adding the weighted spectra of used light sources (e.g. single-colour LEDs) and due to the fact that there is no "negative light". Because of that usual optimization methods like least square method cannot be used. A novel synthesizing method based on a constrained optimization process was developed and tested on the laboratory TCLS to be used for calibration purposes. The developed synthesizing method, described in this paper, gives good results but comparison with more simple methods shows that also these can be successfully used.

Keywords: LED, tuneable colour light source, synthesized spectral power distribution, constrained least squares problem

1. INTRODUCTION

Tuneable colour light sources (TCLS) are suitable for different purposes and can be used in laboratories or in our everyday lives. TCLS are actually multi-channel light sources based on LEDs, which are able to mimic spectral power distributions (SPDs) of different light sources and also different CIE Standard Illuminants, even those which are only defined by mathematical models.

Spectra of modern light sources based on light emitting diodes (LEDs) are much different from spectra of classical light sources. To reduce measurement uncertainty of new LED based products, measurement equipment (photometers) should be calibrated not only with sources providing the spectrum of CIE Standard Illuminant A but also with sources providing a spectrum which is very close to the one intended to be measured. That is why a TCLS can be very useful in a photometric laboratory. As it is designed to simulate any chosen spectrum, it can be used for calibrations of equipment with varying spectra, so reducing the number of different calibration light sources in use in a laboratory. But as it will be used for calibration it is important that such a laboratory TCLS is able to produce various SPDs with defined and stable photometric or colorimetric parameters like luminance, correlated colour temperature (CCT), colour coordinates (x, y), or colour rendering index (CRI).

To be able to utilize the TCLS, described in [1], for calibration and research purposes a new set of LEDs was installed, and a new control program was written. The main purpose of the TCLS will be the calibrations of measurement devices at a series of different spectra, so the TCLS needs to enable fast and precise setting of the wanted spectrum.

The core of the control program is a synthesizing process for TCLS output spectrum. It needs to give the best possible solution; in our case this is an output spectrum which should be as close to the target spectrum as possible and in an acceptably short time. As the TCLS output spectrum needs to be very close to the target spectrum, the synthesizing process was based on an optimization method where the difference between output and target was used as an optimization criterion. The development and testing of the synthesizing process is described below.

2. SYNTHESIZING OF THE SPECTRUM

The idea behind the TCLS based on LEDs was to build a device capable of producing any wanted spectrum in the spectral range covered by the LEDs. The initial version of the TCLS was built with the help of an integrating sphere equipped with 24 different so-called monochromatic LEDs installed in a circle around the output port so as to illuminate the back of the sphere. Although called monochromatic, used LEDs actually emit a narrow bandwidth of light with FWHM (Full width at half maximum) between 15 nm and 35 nm as it can be seen in Fig. 3.

The LEDs are placed so that they cannot be seen directly from the output port. As shown in Fig. 1, the output port of the TCLS is also equipped with a baffle tube containing four apertures to minimize the entrance of stray light from the environment into the sphere. In the sphere, the spectra of LEDs used are mixed by multiple diffuse reflections and the output port provides access to a uniform luminous area approximating the synthesized spectrum. A multi-channel DC power supply is used to control each LED individually by a set current. The DC power supply is connected to the PC via a GPIB bus, which enables the LEDs control by computer program. The sphere also includes a fibre-optic port for a spectroradiometer, which is connected to the same PC. The measured spectral data is processed by the control program written in a LabVIEW environment, which can be used to control and also to regulation of the TCLS output.

There are a lot of articles related to the topic of tuneable colour light sources based on LEDs, and also how to fit the synthesized spectrum as close to the wanted spectrum as possible. Different studies have tried to find the best way to mimic a spectrum of different CIE standard sources and other illuminants. Fryc et al. [2] proposed a tuneable light source using LEDs in the (380–780) nm region, with a simple but slow iterative optimization procedure. Wu et al. [3] introduced a pruning process to achieve the smallest difference between optimized and wanted spectrums by removing improper LEDs, thereby finding an optimal set of LEDs. The results in the paper are very promising, but unfortunately, we were unable to recreate the procedure as not all steps are fully described in the paper. Luo et al. [4] proposed a stochastic radial basis function algorithm for nonlinear optimization of an LED-based spectrum, where a minimization of colour difference equation was included.

Procedures in the mentioned papers are all derived from the Gaussian optimization method, which is based on a minimization of the sum of squared differences between the spectrum synthesized by TCLS and the target spectrum. The least squares solution does indeed give the optimal result, but it can also contain some negative values of coefficients, which cannot be realized in the case of TCLS, because in this case the obtained coefficients actually represent light outputs of LEDs, where negative coefficients would mean negative light output subtracting this from synthesized light which is physically not possible. Therefore, an enhanced or a different optimization method, which would give the best result by taking into consideration that calculated values of coefficients need to be positive or equal to zero, needed to be found.

On the other hand, the Gaussian optimization method which is a basic mathematical procedure for the calculations of best fit solution coefficients – or at least their initial values – is very simple. According to [5], the Gaussian optimization method can also be used to find a constrained solution, but only if we define these constraints correctly and in a proper way. The ability of Gaussian optimization method to solve least squares problems with linear inequality constraints allowed us to base our synthesizing procedure on Gaussian optimization method also.

3. OPTIMIZATION METHODS WITH CONSTRAINTS

A lot of different optimization methods, which take into consideration some sort of constraints, are described in available articles. The one by Tosic and

Frossard [6] presented the main challenges in the research field of dictionary learning for dimensionality reduction. They focused on the development of novel algorithms for building dictionaries of subspaces that provide efficient representations of classes of signals. Since sparsity constraints are the keys for solving dictionary learning problems, they are all based on sparse approximation. Chun et al. [7] tested optimization methods to minimize reconstruction error and the number of LED sources using these sparse coding techniques from Tosic and Frossard [6]. Lawson and Hanson [5] described a procedure of a non-negative least squares (NNLS) problem which proved to be optimal for a non-negative problem with certain inequality constraints. Bro and De Jong [8] proposed a fast non-negativity-constrained least-squares algorithm, which is based on a standard NNLS algorithm in [5]. In some cases, it converges faster, but the basis of the procedure stays very similar. However, Cantarella and Piatek [9] announced a freely available C implementation of a sparse constrained least-squares problem. The code matches the accuracy of Matlab's function lsqnonneg [10], which is again based on method described in [5]. Cantarella's and Piatek's code works much faster than Matlab's function, and it is more suitable for very large problems.

Due to the fact that many articles use the non-negative least squares (NNLS) method for solving different problems, we can assume that this method gives an optimal solution for the non-negativity least squares problem with certain inequality constraints. This is why we also used it for optimization of TCLS output spectra. Below, we first present a brief description of the Gauss least squares optimization algorithm and second, we present an overview of the main algorithm that uses the NNLS method.

3.1. Gauss Optimization Method

The Gauss algorithm is used to solve non-linear least squares problems. The problem is called 'least squares' because we are minimizing the sum of squares of residuals. In case of TCLS the residuals are differences between the obtained output spectrum of TCLS and target spectrum at the observed wavelengths. The output spectrum of TCLS, denoted with $S_o(\lambda)$ is synthesized from M spectra of LEDs, so it can be represented with the help of the following equation:

$$S_o(\lambda) = \sum_{i=1}^{M} k_i \cdot S_i(\lambda), \quad (1)$$

where $S_i(\lambda)$ denotes the SPD of the i-th LED in the set of LEDs, k_i are their synthesis coefficients and M is the number of LEDs in set. The SPDs of used LEDs can be defined at chosen wavelength e.g. at every 1 nm from 380 nm to 780 nm if we want to use TCLS in the visible part of the spectrum only. The sampled target spectrum can be denoted by $S_t(\lambda)$. So the residual function R, which represents sum of squared differences between target and output spectrum can be written as:

$$R = \sum_{\lambda=380}^{780} (S_t(\lambda) - S_o(\lambda))^2 . \quad (2)$$

Taking into consideration equation 1, R can be further written as:

$$R = \sum_{\lambda=380}^{780} \left[S_t(\lambda) - \sum_{i=1}^{M} k_i \cdot S_i(\lambda) \right]^2 , \quad (3)$$

or in matrix form:

$$R = \left\| \mathbf{S_t} - \mathbf{S_{LED}} \mathbf{K} \right\|^2 , \quad (4)$$

where $\mathbf{S_t}$ is a target spectrum in a vector form with values defined at chosen wavelengths, $\mathbf{K}$ is a vector of synthesis coefficients and $\mathbf{S_{LED}}$ is a matrix composed of spectra of used LEDs defined at the same wavelengths as $\mathbf{S_t}$. Taking into consideration the wavelengths between 380 nm and 780 nm with step of 1 nm vectors $\mathbf{K}$ and $\mathbf{S_t}$ have 401 elements and $\mathbf{S_{LED}}$ is a matrix with 401 x M elements like presented below:

$$\mathbf{S_{LED}} = \begin{bmatrix} d_1(1) & d_2(1) & \cdots & d_M(1) \\ d_1(2) & d_2(2) & \cdots & d_M(2) \\ \vdots & \vdots & \ddots & \vdots \\ d_1(401) & d_2(401) & \cdots & d_M(401) \end{bmatrix}, \quad (5)$$

where $d_n(i)$ is the value of spectrum of n-th LED at i-th wavelength.

The expression in (4) represents the over-determined system of 401 linear equations with M unknowns. Based on Gauss and Legendre discovery the solution for **K**, which minimize the expression in (4), can be found by:

$$\mathbf{K} = \left(\mathbf{S}_{\mathbf{LED}}^{\mathbf{T}} \cdot \mathbf{S}_{\mathbf{LED}}\right)^{-1} \cdot \mathbf{S}_{\mathbf{LED}} \cdot \mathbf{S}_{\mathbf{t}}, \qquad (6)$$

where

$$\left(\mathbf{S}_{\mathbf{LED}}^{\mathbf{T}} \cdot \mathbf{S}_{\mathbf{LED}}\right)^{-1} \cdot \mathbf{S}_{\mathbf{LED}} = \mathbf{S}_{\mathbf{LED}}^{+} \qquad (7)$$

is the Moore-Penrose [11, 12, 13] pseudo-inverse of spectra matrix $\mathbf{S}_{\mathbf{LED}}$..

Unfortunately, in our case the synthesis coefficient vector **K** obtained may or may not be the optimal solution. This is, because in general **K** may contain some negative coefficients which would mean that these LEDs need to produce a negative amount of light. As this is not possible, thus calculated optimal output spectrum cannot be realized practically. This is the reason why we need to include additional constraints in our calculation: the optimal output spectrum will be the one with the smallest *R* and only positive synthesis coefficients.

3.2. Algorithm for Non-negative Least Square Problems

The resulting problem is so called non-negative least squares problem (NNLS) and can be in general defined by the statement:

$$\text{minimize } \|\mathbf{Ax} - \mathbf{b}\|, \text{ subject to } \mathbf{x} \geq 0, \qquad (8)$$

where **A** is the *m x n* matrix, where $m \geq n$, **b** is the *m* element data vector, and **x** is the *n* element solution vector. An optimal solution for the set of linear equations $\mathbf{Ax} \approx \mathbf{b}$ must be found, where $\mathbf{x} \geq 0$. In our case the matrix **A** represent the matrix of LEDs spectra ($\mathbf{S}_{\mathbf{LED}}$) of the used (and measured) LEDs, where *n* is the number of LEDs and $m = 401$ is the size of the sampled SPD vector with data at 1 nm step from 380 to 780 nm. The vector **b** represents the target spectrum $\mathbf{S}_{\mathbf{t}}$ and has the same size $m = 401$ and **x** contains the optimal solution, in our case optimal synthesis coefficients **K**.

The NNLS problem can be solved with different algorithms. The first widely used algorithm was described by Lawson and Hanson in [5] and has nine steps. The procedure starts with setting all elements of **x** to zero, creating set **Z**, containing all indices, and empty set **P**. In the main loop the gradient vector **w** is calculated with the current value of **x**, with the equation

$$\mathbf{w} = \mathbf{A}^{\mathbf{T}} (\mathbf{b} - \mathbf{Ax}). \qquad (9)$$

If **Z** is empty or if all elements of **w**, with indices in **Z**, have values ≤ 0, we have a solution, therefore the procedure terminates. Otherwise in the next step the maximum element of **w** is moved from set **Z** to **P**. If any of the elements have negative values, only a fraction of **Z** can be accepted as a trial solution. So we need to find an index *q* such that

$$\alpha = \frac{x_q}{(x_q - z_q)} \qquad (10)$$

is the minimum of all such expressions for negative elements of **Z**. With the expression α for this *q* the linear sum

$$\mathbf{x} = \mathbf{x} + \alpha (\mathbf{z} - \mathbf{x}) \qquad (11)$$

can be calculated. In the last step all indices for which the corresponding elements of *x* is zero, are moved from set **P** to **Z**. These will include x_q, but may also include other elements as well. When the procedure converges, set **P** is a vector of synthesis coefficients.

At the end not all elements of *x* are positive and in **P**. Some of them are left in **Z** with the value of 0. In the case of TCLS that means not all LEDs will contribute to the synthesized output spectrum. The needed ones will be supplied with the proper currents based on values of positive synthesis coefficients in **P** (or in **K** in our case) and the other will be switched off as their synthesis coefficients have value 0.

3.3. Other Simpler Methods to Get the Optimal Solution

The described mathematical procedure is not very complicated but might cause some problems if we would need to implement it in some simple TCLS, e.g. controlled with microcontroller. Even when it was implemented within the LabVIEW program to control the TCLS, we had some difficulties and the control program was rather slow. That's why we also tested some simpler "optimization" procedures.

These additional procedures were designed to be easier to implement in the LabVIEW or even microcontroller environment. To shorten the calculation time, the Gauss optimization method was used as a starting point. As described in 3.1, the main problem of Gauss optimization procedure is that the vector of optimal coefficients that we get with equation (6) can contain some negative values, which cannot be realized practically. Hence, the first approach is to exclude the LEDs with negative coefficients from the used set of LEDs and synthesize the output spectrum only with LEDs with positive coefficients. This is a very fast and simple method but the results are in most cases not very good. As we use only a small number of LEDs to synthesize the target spectrum the residual *R* (equation 4) is in most cases rather large.

Beside this basic method, which excludes all LEDs with negative values at once (no iterations), we also tested four other methods, which exclude LEDs with negative values step by step until only LEDs with positive values are left in the set and Gauss optimization procedure gives only positive synthesis coefficients. The three tested procedures of excluding LEDs with negative values step by step differ only in the way of excluding the first LED with a negative value. In the first procedure we excluded first the LED with the most negative value of synthesis coefficient. In the second procedure, the LED, which was excluded first, was the one with the least negative synthesis coefficient. In the third and fourth procedure we just excluded the first (last in fourth procedure) LED on a list with the negative synthesis coefficient. At the end of the iterative process when Gauss optimization procedure results in only positive synthesis coefficients, a synthesized output spectrum can be calculated with equation (1), where only LEDs with positive synthesized coefficients are used.

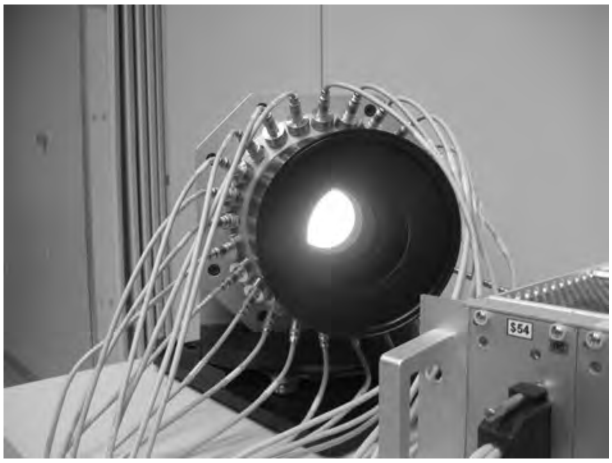

Fig.1. Realization of a tuneable colour light source based on integrating sphere and 24 LED placed around the output port

3.4. Comparison of Optimization Methods

All described methods were tested with the laboratory TCLS and with help of two different sets of LEDs. Since the size of the sphere is limited, the number of LEDs in one set is restricted to 24. The LEDs in the first set were all monochromatic and chosen so that their SPDs would be as evenly distributed throughout the whole visible spectrum range from 380 nm to 780 nm as possible. Such a distribution gives a continuous synthesized spectrum which could, at least in principle, be closer to the different target spectra. If coefficients of some LEDs are zero, when synthesizing target spectrum, currents of these LED's are set to zero. Therefore,

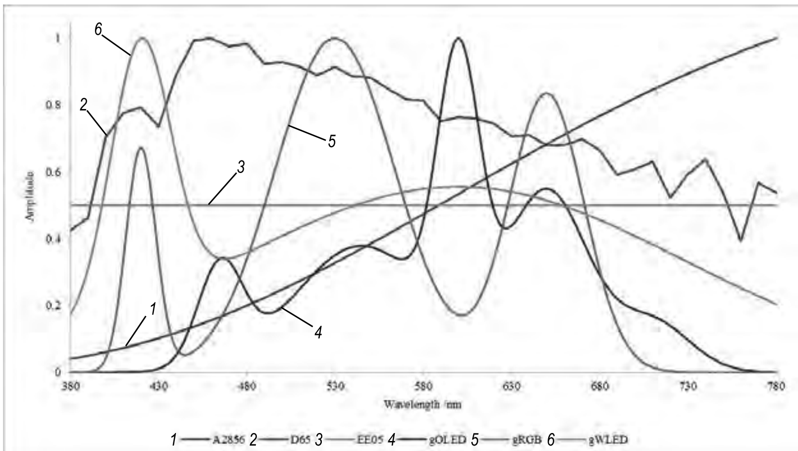

Fig.2. Six different target (wanted) spectra used for testing the TCLS optimization methods

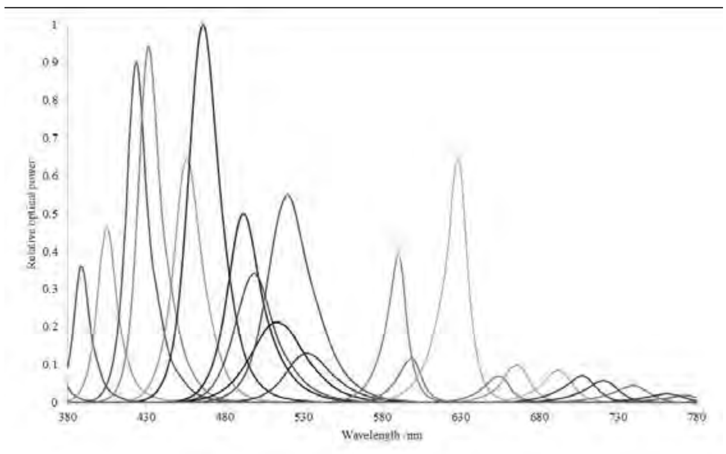

Fig.3. Relative spectra of 24 LEDs used in TCLS, measured at the nominal current of LEDs. Spectra were scaled so that the highest measured peak has a value of 1

these LEDs are not lit up but they stay mounted in TCLS. This is useful, when we want to use the TCLS for synthesizing more than one wanted spectrums. Hence, one LED might be turned off for one optimized spectrum, but is turned on in another synthesized spectrum. If calculations show, that one of the coefficients equals zero in all wanted spectrums, this LED can then be removed from the LED set, since it doesn't impact any of the wanted spectra. In such a case, a new LED can be installed in TCLS. Also in our first realization of TCLS one such LED emerged and it was later replaced with a white LED.

For first tests TCLS was equipped with 24 monochromatic LEDs. We tested it with six different target (wanted) spectra from various types of light sources: CIE standard illuminant A (T_{cc}= 2856 K) and D65 sources, an equal energy source (EE05), a generic OLED source (gOLED), a generic RGB source (gRGB), and a generic cold white LED source (gWLED). The used relative target spectra can be seen in Fig. 2. The spectra of the selected LEDs in the first set with 24 monochromatic LEDs are shown in Fig. 3.

As can be seen from Fig. 3 there is a lack of appropriate LEDs with peak wavelengths (WL) in the "green" region between 520 nm and 590 nm. As expected this caused rather significant deviation in synthesized spectrum from the target spectrum in that specific area.

In Table 1, the relative power of the individual LEDs used in set 1 is listed in the third and sixth columns to show different power outputs of the LEDs. Spectra were scaled according to the one with the maximum power output. Since LEDs with peak WL from 650 nm to 780 nm have rather low output compared to other LEDs, the maximal total luminous flux output of the TCLS may also be very low for some synthesized spectra. The first measu-

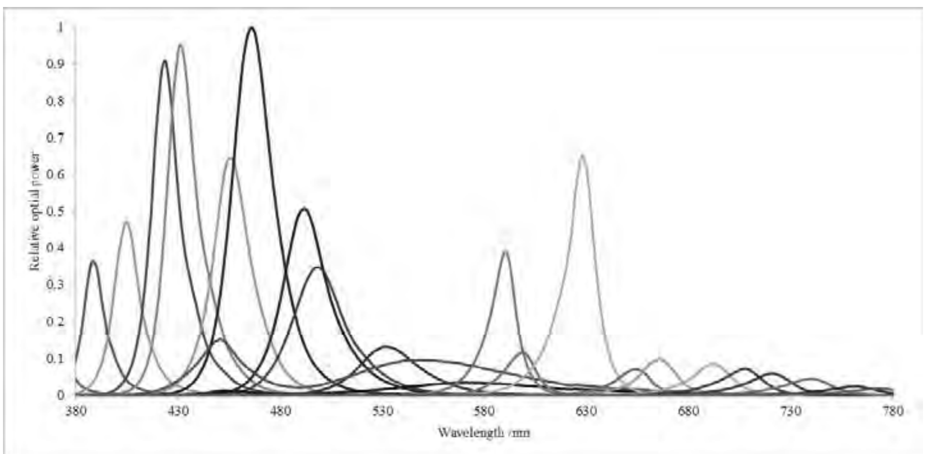

Fig.4. Relative spectra of 24 LEDs in set 2, where two monochromatic LEDs were replaced by warm white and cold white LEDs

Table 1. Peak WL (nm) and Relative Power of Chosen 24 LEDs

LED	Peak WL /nm	Relative peak power	LED	Peak WL /nm	Relative peak power
1	380	0,042	13	590	0,390
2	388	0,361	14	599	0,116
3	405	0,463	15	628	0,648
4	424	0,900	16	654	0,069
5	431	0,943	17	666	0,099
6	456	0,645	18	692	0,086
7	466	1	19	707	0,071
8	492	0,500	20	721	0,058
9	498	0,341	21	739	0,044
10	513	0,213	22	762	0,023
11	531	0,131	23	774	0,018
12	520	0,550	24	780	0,003

Table 2. Peak WL (nm) and Relative Power of White LEDs

LED	CCT /K	Peak WL /nm	Relative peak power
Warm white	3500	572	0.0339
Cool white	8700	451	0.1516

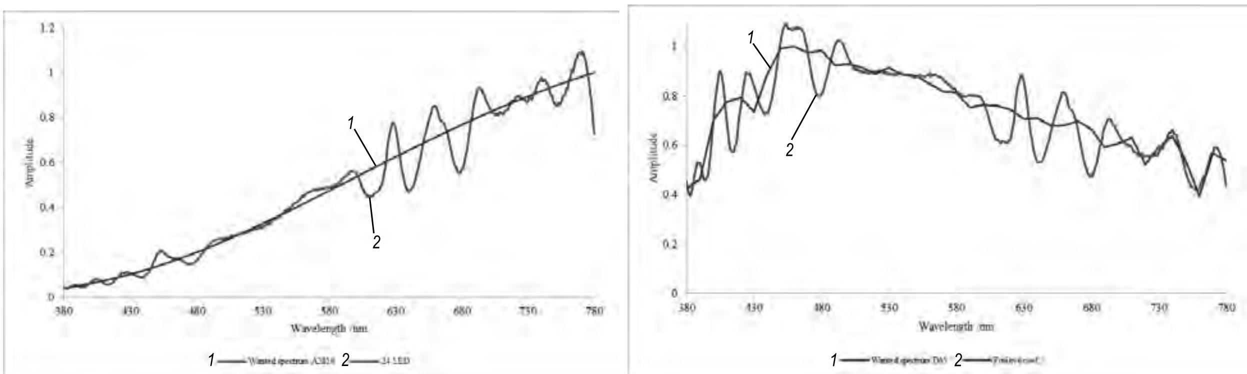

Fig.5. Results of synthesis of CIE Illuminant A spectrum (left) and D65 spectrum (right)

rements showed that the peak value of the 24th LED is very small, and with its peak WL of 780 nm it is also practically out of the visual spectrum. As the total luminous flux of the TCLS depends not only on the chosen target spectrum but also on peak powers of the used LEDs, the mentioned 24th LED always causes a rather low total luminous flux output. Therefore, this LED was not included in most of the tests and was later replaced.

To minimize the deviations from the target spectrum in the range between 520 nm and 590 nm, it was found out during the initial tests that two white LEDs (a warm white LED and cool white LED) will improve the spectrum. Hence, two phosphor-converted white LEDs, one cold-white and one warm white, were added to the set in the second step (Table 2). But because of the limited number of places for LEDs in the sphere, two existing LEDs had to be removed from the set. We removed LED no. 12, whose coefficient was zero with all synthesized test spectra and the LED no. 24, which was not included in most synthesized spectra due to its

Table 3. Comparison of Results of Used Methods with Two LED Sets – the Numbers in the Table are Calculated Residual Functions (*R*)

Synthesized Spectrum	A2856	D65	EE05	gOLED	gRGB	gWLED
Set with 23 monochromatic LEDs (No. 24 not used)						
NNLS	5,6745	15,725	5,847	3,9593	5,805	7,23419
all neg. to zero	132,94	868,42	262,0	92,380	278,2	252,666
most neg. first	5,6745	15,725	5,847	3,9593	5,805	7,23419
least neg. first	5,6824	15,839	5,888	4,1399	5,993	7,26875
first neg. first	5,6824	15,839	5,888	3,7566	5,206	7,26875
last neg. first	5,6745	15,725	5,847	3,9593	5,805	7,23419
Set with 22 monochromatic LEDs and two white LEDs						
NNLS	1,6819	2,6235	1,118	1,3999	2,916	1,71788
all neg. to zero	2,8366	9,5895	3,235	3,9093	6,052	4,28758
most neg. first	1,6819	2,6499	1,124	1,3999	2,916	1,72942
least neg. first	1,6837	2,8188	1,227	1,4000	2,926	1,85828
first neg. first	1,6819	2,6235	1,118	1,3999	2,916	1,71788
last neg. first	1.6837	2.6499	1.124	1.3999	2.916	1.72942

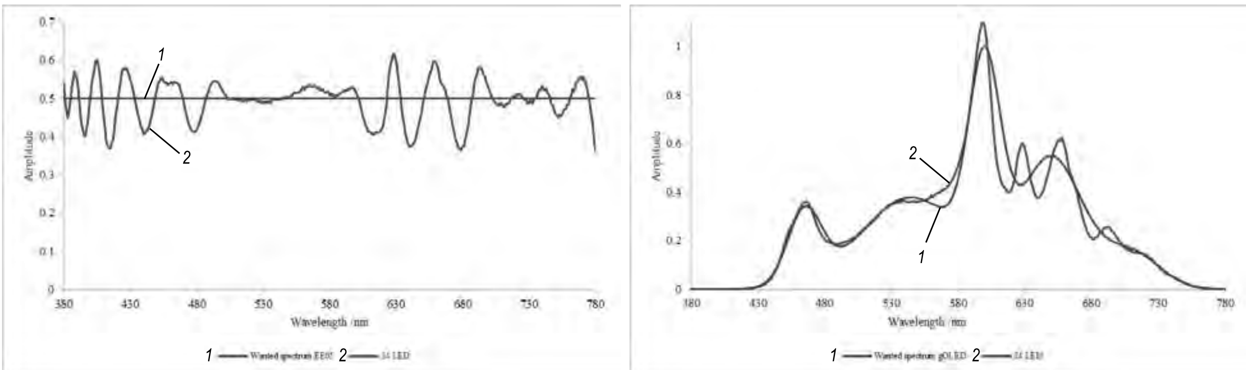

Fig.6. Synthesizes spectra of "equal energy" source (left) and generic OLED source (right)

very low light output. As shown in Fig. 4, the chosen two white LEDs do cover the area of the spectrum between 520 nm and 590 nm.

All described optimization methods were tested with both sets of LEDs. The spectra shown in Fig.2 were used as test target spectra. The comparison of the result is shown in Table 3. The NNLS method serves to find the smallest difference (smallest residual function R) between the synthesized and target spectrum for both LED sets, and hence, represents their best results. With both LED sets the results of at least one other (computing time improved) method (using different expelled LEDs) is equal to the results of the NNLS method. In the first LED set the best results are also given by two of the computing time improved methods, namely the method of excluding the LED with the most negative value first and the method of excluding the last LED with the negative value first. The method that also gives the best results with the second LED set is the method of excluding the first LED with negative value first. Other methods do not give comparable results.

4. RESULTS

The grey cells in Table 3 show the best results with the smallest obtained residual functions. On the basis of the results obtained in the tests, the above described method of NNLS does give the best results and therefore, the smallest deviations

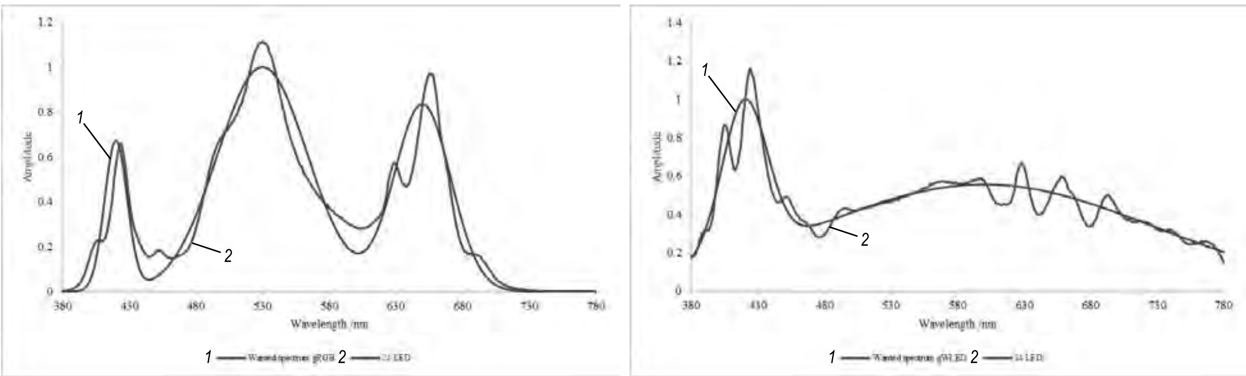

Fig.7. Results of synthesis of generic RGB LED (left) and generic white LED (right)

from the target spectrum with both LED sets. Only the method where all LEDs with negative values are excluded at once after first calculation of synthesis coefficients does not give comparable results, as the values of the residual functions are much higher in all synthesized spectrums. Beside NNLS method also the computing time improved optimization methods can be used equally, as the obtained results are the same or very close in most of cases. In particular, the method, where the LED with the most negative value is excluded first, is very promising. It's much faster and gives results almost all of which match those of the NNLS method

In the Figs. 5–7, the resulted optimized spectra are shown. They are all obtained with the NNLS optimization procedure and with the second set of LEDs containing 22 monochromatic LEDs and two white LEDs.

5. CONCLUSION

The aim of the research described in this paper is to find an optimal mathematical method which gives the synthesized spectrum of multiple LEDs light source as close to the wanted spectrum as possible. Based on literature review an optimization method for finding an optimal solution for the non-negativity least squares problem with certain inequality constraints was tested together with some more simple optimization methods. For the test purposes two different sets of LEDs were used to synthesize six target spectra. The described optimization procedure based on non-negative least square algorithm (NNLS) appears to be very useful for the tested setup, moreover it gives the best results for both tested sets of LEDs. Surprisingly, the results obtained with the much simpler methods, described in chapter 3.3. gave in most cases the same or very similar results compared to the sophisticated NNLS procedure. As expected, the only method with much worse results was the one, where already in the first optimization step all LEDs with the negative synthesis coefficients were taken out of the active set.

Despite the complexity of the NNLS procedure, it is the most adaptable tool for a laboratory tunable colour light source (TCLS) based on integrating sphere and controlled by a LabVIEW environment. However, due to the non-linearity of the current-dependent LED output characteristic which in addition depends on the temporal variation of the junction temperature of the LEDs, the realization of such a controlled TCLS usable for calibration is not so straight-forward as may be expected and it places some demands on the robustness of the NNLS procedure.

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