*Light & Engineering*
*Vol. 28, No. 1, pp. 108–115, 2020*
*https://doi.org/10.33383/2019-023*
*Svetotekhnika #5, 2019, pp. 84–89*

# THE IMPACT OF LIGHT POLARISATION ON LIGHT FIELD OF SCENES WITH MULTIPLE REFLECTIONS

Vladimir P. Budak and Anton V. Grimailo

*NRU MPEI, Moscow*
*E-mail: budakvp@gmail.com; grimailoav@gmail.com*

## ABSTRACT

The article describes the role of polarisation in calculation of multiple reflections. A mathematical model of multiple reflections based on the Stokes vector for beam description and Mueller matrices for description of surface properties is presented. On the basis of this model, the global illumination equation is generalised for the polarisation case and is resolved into volume integration. This allows us to obtain an expression for the Monte Carlo method local estimates and to use them for evaluation of light distribution in the scene with consideration of polarisation. The obtained mathematical model was implemented in the software environment using the example of a scene with its surfaces having both diffuse and regular components of reflection. The results presented in the article show that the calculation difference may reach 30 % when polarisation is taken into consideration as compared to standard modelling.

**Keywords:** polarisation, multiple reflections, local estimates, Monte Carlo method, Mueller matrix, Stokes vector

## INTRODUCTION

The development of computers and software over the previous decades has led to the fact that nowadays design of any lighting installation involves modelling light field of illumination scenes, which will be obtained when using the selected lighting devices. Calculation of multiple reflections (MR) of light from the surfaces of the scene being modelled plays a very important role in it.

In lighting engineering, it is common to neglect light polarisation phenomena in calculations. Such neglect does not lead to a significant error in the results when it comes to a small number of reflections from surfaces with mostly diffuse nature of reflection. However, if it is necessary to deal with surfaces where reflection has a significant regular component, the state of polarisation of even completely depolarised light will be changed already after the first reflection, which will affect the nature of further interactions of light with the surfaces of the scene.

It is obvious that the light will be depolarised again after a sufficient number of reflections. Nevertheless, it is still unknown how consideration of polarisation will affect the final result of lighting calculation. The existing estimations indicate that the difference between the results of conventional calculation and those of calculation considering light polarisation may exceed 20 % [1]. Once this assumption is confirmed it will mean the necessity to take the state of polarisation into account for solving applied problems (such as calculation of illuminance in the premises with consideration of MR).

The results of a large number of studies by different authors [2–7], which show the sufficient effect of polarisation on the obtained images of scenes when modelling light distribution using their example, have been published recently. However, these studies do not contain information on how consideration of polarisation affects the obtained values of magnitudes describing the energy performance of light radiation. At the same time, it is these magnitudes that interest most the specialists whose work is related to solving practical problems.

Moreover, the images of scenes presented in the works by the said authors show that the form and location of glares on the scene surfaces change when images are rendered with consideration of polarisation. Therefore, the state of light polarisation ultimately affects not only quantitative but also qualitative characteristics of light distribution created by a given lighting installation.

## 1. PHOTOMETRIC DESCRIPTION OF LIGHT POLARISATION

To select the method of description of light polarisation, it is necessary to pay attention to the fact that all photometric terms are formulated exceptionally as *observable* values. The nature of these values is determined, in particular, by quadratic characteristics of optical radiation detectors (i.e. response to power), finitude of their dimensions and time constant [8].

In its turn, the electromagnetic field theory uses such values which *are impossible to measure directly by experiment:* amplitude and wave field phase. Therefore, when describing any wave optics experiment, the necessity to transfer to the photometric interpretation of light field is inevitable [9].

In the authors' opinion, it means that the language of polarisation description which is the most corresponding to the processes of radiation measurement by an optical detector includes application of a set of four parameters (or, in other words, the four-dimensional parameter vector) introduced by G.G. Stokes in 1852 in [10]. These parameters describe light such that, if any beams obtained independently of each other have the same values of all four components of the parameter vector, they are optically equivalent and no experiment allows distinguishing them [10].

The works [8, 9, 11] show that for purposes of photometry the Stokes parameters have dimensions of *luminance* which, in its turn, fully characterises radiation. Therefore, the full description of a beam shall include a set of four parameters.

It is possible to determine the components of the Stokes parameter vector based on the electromagnetic theory, like it is done in [11], or experimentally by passing the radiation through a set of polarisation filters [12]. In this case, the said parameters are determined based on reactions $J_i$ of the corresponding optical detectors:

$$L_0 = 2J_0, \quad L_1 = 2(J_0 - J_1), \\ L_2 = 2(J_2 - J_0), \quad L_3 = 2(J_3 - J_0), \tag{1}$$

where $J_i, i \in \overline{0,3}$, are differ the installed polarisation filters:

$J_0$ is a neutral filter with transmission of 0.5;

$J_1$ is an analyser, with its optical axis and the direction of radiation distribution determine the coordinate system – a reference plane;

$J_2$ is an analyser with axis angle of 45° to the reference plane;

$J_3$ is a complex filter consisting of a quarter-wave plate and an analyser with angle of 45° to the reference plane.

It should be highlighted that the most important characteristic of the Stokes parameters is the coordinate system in which they are defined or the reference plane; horizontal and vertical positions are defined relative to this plane. The components $L_1$ and $L_2$ depend on selection of the plane while $L_0$ and $L_3$ don't [12].

For each interaction of light and the environment, the parameters are calculated relative to a new reference plane which is linked with the diffuse point and is obtained by turning the previous plane about a corresponding angle.

Let us assume that the reference plane $\zeta$ was obtained by turning the previous plane $\zeta'$ about some angle $\varphi$ relative to the $Z$ axis. Then the following expression is fulfilled for the Stokes vectors $\mathbf{L} = \{L_0, L_1, L_2, L_3\}$ and $\mathbf{L}' = \{L_0', L_1', L_2', L_3'\}$ defined relative to $\zeta$ and $\zeta'$ [13]:

$$\mathbf{L} = \vec{\mathrm{R}}\,\mathbf{L}', \tag{2}$$

where $\mathbf{L}'$ is the Stokes vector *before* interaction; $\mathbf{L}$ is the Stokes vector *after* interaction; $\vec{\mathrm{R}}$ is the reference plane rotation matrix or rotator (from the Latin word "rotatio") which is defined as follows [13]:

$$\vec{\mathrm{R}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\varphi & \sin 2\varphi & 0 \\ 0 & -\sin 2\varphi & \cos 2\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

The sign of $\varphi$ is defined based on the condition that the coordinate system associated with the beam is right-handed: a positive value of angle $\varphi$ corresponds to an anti-clockwise turn if looking from the side of positive values of the axis $Z$.

It is worth noting that the following system of indications is used hereinafter:

$\mathbf{a}$, $\vec{\mathbf{a}}$, $\vec{\mathrm{a}}$, $\vec{a}$ is the column vector;

$\overleftarrow{\mathbf{a}}$, $\overleftarrow{\mathrm{a}}$, $\overleftarrow{a}$ is the row vector;

$\hat{\mathbf{a}}$, $\hat{\mathrm{a}}$, $\hat{a}$ is the unit column vector;

$\overleftrightarrow{\mathbf{a}}$, $\overleftrightarrow{\mathrm{a}}$, $\overleftrightarrow{a}$ is the matrix;

$\mathbf{a}\times\mathbf{b}$ is the plane generated by vectors $\mathbf{a}$ and $\mathbf{b}$ with a normal of $\hat{\mathbf{N}} = \dfrac{[\mathbf{a}\times\mathbf{b}]}{|[\mathbf{a}\times\mathbf{b}]|}$.

## 2. MATHEMATICAL MODEL OF MULTIPLE REFLECTIONS OF LIGHT WITH CONSIDERATION OF POLARISATION

In order to create a mathematical model considering light polarisation, the authors used the method proposed by G.V. Rosenberg in [11] which is based on use of the Stokes parameter vector for description of the state of a light beam and Mueller matrices for description of light interaction with a substance. Let us review this method in detail.

Distributing in some medium, a beam interacts with substance. In cases when electrodynamics equations are linear and homogeneous, the result of such interaction may be presented in the following form:

$$\mathbf{L}(\mathbf{r},\hat{\mathbf{l}}) = \overleftrightarrow{\mathrm{R}}(\hat{\mathbf{l}}'\times\hat{\mathbf{l}},\hat{\mathbf{N}}\times\hat{\mathbf{l}})\cdot \\ \cdot\overleftrightarrow{\rho}(\mathbf{r},\hat{\mathbf{l}},\hat{\mathbf{l}}')\overleftrightarrow{\mathrm{R}}(\hat{\mathbf{l}}'\times\hat{\mathbf{N}}',\hat{\mathbf{l}}'\times\hat{\mathbf{l}})\,\mathbf{L}(\mathbf{r},\hat{\mathbf{l}}'), \tag{4}$$

where $\overleftrightarrow{\rho}(\mathbf{r},\hat{\mathbf{l}},\hat{\mathbf{l}}')$ is the 4×4 Mueller matrix which describes the effect on the beam caused by the substance; $\overleftrightarrow{\mathrm{R}}(\hat{\mathbf{l}}'\times\hat{\mathbf{N}}',\hat{\mathbf{l}}'\times\hat{\mathbf{l}})$ is the matrix of the reference plane rotation from $\hat{\mathbf{l}}'\times\hat{\mathbf{N}}'$ to $\hat{\mathbf{l}}'\times\hat{\mathbf{l}}$. The plane $\hat{\mathbf{l}}'\times\hat{\mathbf{N}}'$ is generated by direction of the beam after the *previous* diffusion and normal of the element of the surface on which the *previous* interaction occurred; $\hat{\mathbf{l}}'\times\hat{\mathbf{l}}$ is generated by directions of the beams after *the previous and the current* interactions; $\hat{\mathbf{N}}\times\hat{\mathbf{l}}$ is generated by the direction of the beam after the *current* interaction and the normal of the element of the surface on which the *current* interaction occurred.

The result of a row of transformations will be obtained by applying the corresponding matrix $\overleftrightarrow{\rho}$, which is the product of matrices of partial transformations:

$$\overleftrightarrow{\rho} = \prod \overleftrightarrow{\mathrm{R}}_i\,\overleftrightarrow{\rho}_i\,\overleftrightarrow{\mathrm{R}}'_i. \tag{5}$$

This work studied only the case of light reflection from scene surfaces with different fraction of Fresnel component. That is why below the construction of Mueller matrices which describe change of the Stokes vector only at the interface of media with different refractive indexes is presented.

Let us assume $\theta$ is the angle of incidence, i.e. the angle between the direction $-\hat{\mathbf{l}}'$ and the vector $\hat{\mathbf{N}}$ of normal to the surface (Fig. 1); $\theta_{\mathrm{reflec}}$ is the angle of reflection; $\theta_{refrac}$ is the angle of refraction.

Direction $\hat{\mathbf{l}}$ of the beam reflected from the interface will be determined as:

$$\hat{\mathbf{l}} = \hat{\mathbf{l}}' - 2(\hat{\mathbf{N}},\hat{\mathbf{l}}')\hat{\mathbf{N}}. \tag{6}$$

Generally, for reflection from the border of two dielectric media, the Mueller matrix $\overleftrightarrow{\rho}$ has the following form [13]:

$$\overleftrightarrow{\rho} = \frac{1}{2}\begin{pmatrix} \rho_\perp + \rho_\parallel & \rho_\perp - \rho_\parallel & 0 & 0 \\ \rho_\perp - \rho_\parallel & \rho_\perp + \rho_\parallel & 0 & 0 \\ 0 & 0 & 2\sqrt{\rho_\perp\rho_\parallel} & 0 \\ 0 & 0 & 0 & 2\sqrt{\rho_\perp\rho_\parallel} \end{pmatrix}, \tag{7}$$

where $\rho_\parallel = \dfrac{\mathrm{tg}^2(\theta-\theta_{refrac})}{\mathrm{tg}^2(\theta+\theta_{refrac})}$ is the Fresnel reflection coefficient for a beam linearly polarised in the in-

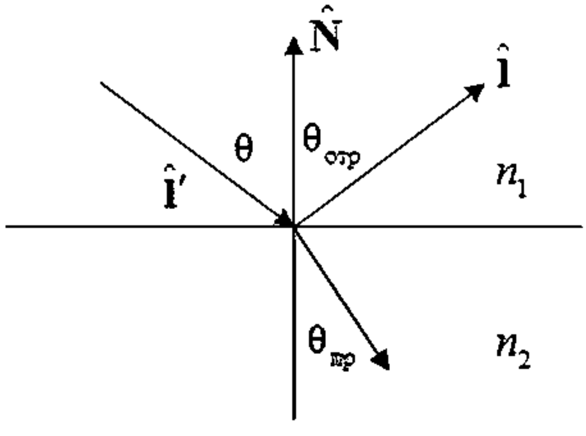


Fig. 1. Beam path at the interface of two media

cidence plane; $\rho_{\perp} = \frac{\sin^2(\theta - \theta_{refrac})}{\sin^2(\theta + \theta_{refrac})}$ is the Fresnel reflection coefficient for a beam linearly polarised perpendicularly to the incidence plane.

Both the reflected wave and the refracted wave retain polarisation they had before interaction.

In the particular case of normal incidence $(\theta = 0)$, when the reflection coefficient does not depend on polarisation, the matrix $\vec{\rho}$ will be as follows:

$$\vec{\rho} = \rho \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (8)$$

For the Brewster angle $\theta = \theta_B$, with $R_{\parallel} = 0$, the matrix is equal to:

$$\vec{\rho} = \frac{1}{2} \cos^2 2\theta_B \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \text{ and}$$

$$\mathbf{L}_{ref} = \frac{1}{2}(L_0 + L_1) \cos^2 2\theta_B \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}. \quad (9)$$

Drawing an analogy between luminance and the Stokes vector (which is the "vector luminance"), it is possible to obtain the global illumination equation with consideration of polarisation similar to [14, 5].

With consideration of (2) and (4), the relation between the incident radiation and diffused radiation will be defined in the following way:

$$\mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}) = \frac{1}{\pi} \int \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{l}}, \hat{\mathbf{N}} \times \hat{\mathbf{l}}) \vec{\rho}(\mathbf{r}, \hat{\mathbf{l}}, \hat{\mathbf{l}}') \cdot \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{N}}', \hat{\mathbf{l}}' \times \hat{\mathbf{l}}) \cdot \mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}') \, |(\hat{\mathbf{N}}, \hat{\mathbf{l}}')| \, d\hat{\mathbf{l}}', \quad (10)$$

where $\hat{\mathbf{l}}'$ is the unit vector of radiation incidence direction; $\hat{\mathbf{l}}$ is the same for diffusion; $\mathbf{r}$ is the radius vector of the diffusion point; $\mathbf{L}(\mathbf{r}, \hat{\mathbf{l}})$ is the Stokes vector at point $\mathbf{r}$ along the direction $\hat{\mathbf{l}}$; $\hat{\mathbf{N}}$ is the normal to the surface; $\vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{N}}, \hat{\mathbf{l}}' \times \hat{\mathbf{l}})$ is the matrix of rotation of the reference plane from $\hat{\mathbf{l}}' \times \hat{\mathbf{N}}$ to $\hat{\mathbf{l}}' \times \hat{\mathbf{l}}$; $\vec{\rho}(\mathbf{r}, \hat{\mathbf{l}}, \hat{\mathbf{l}}')$ is the Mueller matrix at reflection point with the specified directions of incidence and diffusion of radiation.

Let us assume that there is no absorption, diffusion and refraction in the medium between the surfaces of the scene. Then we have the boundary value problem of the radiative transfer equation (RTE):

$$(\nabla, \hat{\mathbf{l}}) \, \mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}) = 0, \quad (11)$$

with boundary conditions on the diffusing surfaces:

$$\mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}) = \frac{1}{\pi} \oint \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{l}}, \hat{\mathbf{N}} \times \hat{\mathbf{l}}) \vec{\rho}(\mathbf{r}, \hat{\mathbf{l}}, \hat{\mathbf{l}}') \cdot \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{N}}', \hat{\mathbf{l}}' \times \hat{\mathbf{l}}) \cdot \mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}') \, |(\hat{\mathbf{N}}, \hat{\mathbf{l}}')| \, d\hat{\mathbf{l}}', \quad (12)$$

and on the radiant surfaces:

$$\mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}) = \mathbf{L}_0(\mathbf{r}, \hat{\mathbf{l}}). \quad (13)$$

Solving the equation (1) and proceeding to the surface integral, we will obtain:

$$\mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}) = \mathbf{L}_0(\mathbf{r}, \hat{\mathbf{l}}) + \frac{1}{\pi} \int \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{l}}, \hat{\mathbf{N}} \times \hat{\mathbf{l}}) \vec{\rho}(\mathbf{r}, \hat{\mathbf{l}}, \hat{\mathbf{l}}') \cdot \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{N}}', \hat{\mathbf{l}}' \times \hat{\mathbf{l}}) \mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}') \cdot \frac{|(\hat{\mathbf{N}}, \hat{\mathbf{l}}')(\hat{\mathbf{N}}', \hat{\mathbf{l}}')|}{(\mathbf{r} - \mathbf{r}')^2} d^2\mathbf{r}', \quad (14)$$

where $\hat{\mathbf{N}}' = \hat{\mathbf{N}}(\mathbf{r}')$ is the normal to the surface at point $\mathbf{r}'$. The integral is taken through the directly visible part of the scene surface.

The global illumination equation (GIE) for the Stokes vector will be written in the final form if the visibility function $\Theta(\mathbf{r}, \mathbf{r}')$ of the element $d^2\mathbf{r}'$ from the point $\mathbf{r}$ is introduced in it:

$$\mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}) = \mathbf{L}_0(\mathbf{r}, \hat{\mathbf{l}}) + \frac{1}{\pi} \int \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{l}}, \hat{\mathbf{N}} \times \hat{\mathbf{l}}) \vec{\rho}(\mathbf{r}, \hat{\mathbf{l}}, \hat{\mathbf{l}}') \cdot \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{N}}', \hat{\mathbf{l}}' \times \hat{\mathbf{l}}) \mathbf{L}(\mathbf{r}, \hat{\mathbf{l}}') \cdot F(\mathbf{r}, \mathbf{r}') \Theta(\mathbf{r}, \mathbf{r}') d^2\mathbf{r}', \quad (15)$$

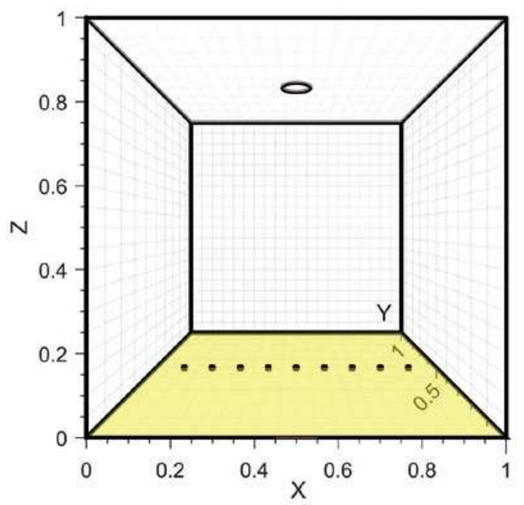


Fig. 2. Scene for modelling multiple reflections

$$F(\mathbf{r},\mathbf{r}')=\frac{|(\hat{\mathbf{N}},\hat{\mathbf{l}}')(\hat{\mathbf{N}}',\hat{\mathbf{l}}')|}{(\mathbf{r}-\mathbf{r}')^2}=$$
$$=\frac{|(\hat{\mathbf{N}},\mathbf{r}-\mathbf{r}')(\hat{\mathbf{N}}',\mathbf{r}-\mathbf{r}')|}{(\mathbf{r}-\mathbf{r}')^4}. \quad (16)$$

## 3. SOLUTION OF GLOBAL ILLUMINATION EQUATION

Like the standard global illumination equation obtained in [14, 15], generally, the equation (2) does not have an analytical solution. That is why numerical technique shall be used for its solution. The Monte Carlo method (MCM) is the most commonly used technique for this purpose. This approach is based on finding a solution of a problem by estimating its mathematical expectation by means of modelling random values.

Within the framework of this study, a number of programmes, which implemented the method of direct modelling of light distribution without consideration of polarisation, were developed. In the course of the work, this approach demonstrated a number of its well-known disadvantages related to complication of mesh formation and high memory consumption, therefore, it was considered inefficient for solving the problems of modelling multiple reflections with consideration of polarisation.

For solution of GIE with consideration of polarisation, it was proposed to use local estimates of the Monte Carlo Method. This method was first proposed in [16] and is different from direct modelling methods as in this case we estimated not distribution of photons over *all surfaces* of the scene at once but probability of photons getting exactly into *the points of interest to us.* With respect to the considered problem, the method is based on transfer from the *surface* integral to the *volume* integral by introducing the δ-function under the integral, which allows modelling to construct based on the beam.

Local estimates are widely spread for solution of problems related to radiation transmission in turbid media. When it comes to modelling light distribution during design of lighting installations, this method came into use quite recently and was described, in particular, in [17]. In the same work, it is shown that local estimations allowing to conduct physically adequate modelling of GIE and allow us to estimate all points of interest based on one beam, which increases efficiency of calculations averagely by (80–90) times, as exemplified by solution of the Sobolev problem, [17]. In view of this, application of MCM local estimates for calculations of multiple reflections with consideration of polarisation appears to be even more promising if it is taken into account that it is necessary to perform more operations at each step of the algorithm as compared to standard modelling.

However, the equation (2) is not convenient for application of statistical modelling due to the fact that the required function under the integral stands at point $\mathbf{r}'$ but is determined at point $\mathbf{r}$. To be able to use local estimates, it is necessary to transform this equation. It is also necessary to take into consideration that $\mathbf{r}'$ and $\hat{\mathbf{l}}'$ are not independent and are related to each other in the following way:

$$\hat{\mathbf{l}}'=\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}. \quad (17)$$

Then the equation (2) takes on the following form:

$$\begin{aligned}\mathbf{L}(\mathbf{r},\hat{\mathbf{l}})&=\mathbf{L}_0(\mathbf{r},\hat{\mathbf{l}})+\\&+\frac{1}{\pi}\int\vec{\mathrm{R}}(\hat{\mathbf{l}}'\times\hat{\mathbf{l}},\hat{\mathbf{N}}\times\hat{\mathbf{l}})\vec{\rho}(\mathbf{r},\hat{\mathbf{l}},\hat{\mathbf{l}}')\cdot\\&\cdot\vec{\mathrm{R}}(\hat{\mathbf{l}}'\times\hat{\mathbf{N}}',\hat{\mathbf{l}}'\times\hat{\mathbf{l}})\mathbf{L}(\mathbf{r},\hat{\mathbf{l}}')\delta(\hat{\mathbf{l}}-\hat{\mathbf{l}}')\cdot\\&\cdot F(\mathbf{r},\mathbf{r}')\Theta(\mathbf{r},\mathbf{r}')d^3\mathbf{r}'.\end{aligned} \quad (18)$$

GIE for the Stokes vector contains the δ-function which complicates modelling by the Monte Carlo Method estimates. This aspect may be eliminated by means of spatial integration. As a result, the estimate will take on the following form:

$$\mathbf{I}_\varphi=M\sum_{n=0}^{\infty}\vec{\mathrm{k}}(\mathbf{r},\mathbf{r}')\mathbf{Q}_n \text{ or}$$

$$\begin{pmatrix} I_{\varphi 0} \\ I_{\varphi 1} \\ I_{\varphi 2} \\ I_{\varphi 3} \end{pmatrix} = M \sum_{n=0}^{\infty} \vec{\mathrm{k}}(\mathbf{r},\mathbf{r}') \begin{pmatrix} Q_{n0} \\ Q_{n1} \\ Q_{n2} \\ Q_{n3} \end{pmatrix}, \tag{19}$$

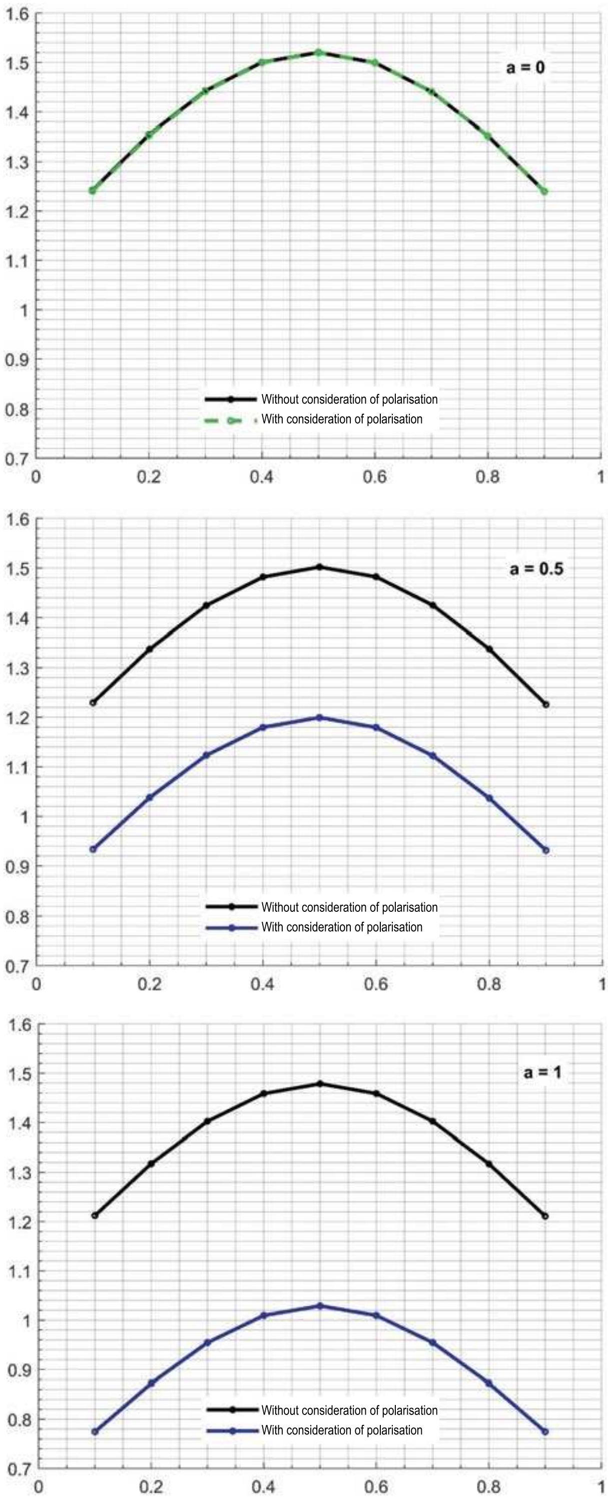


Fig. 3. Distribution of illuminance with different values of *a*

$$\vec{\mathrm{k}}(\mathbf{r},\mathbf{r}') = \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{l}}, \hat{\mathbf{N}} \times \hat{\mathbf{l}})\vec{\rho}(\mathbf{r},\hat{\mathbf{l}},\hat{\mathbf{l}}') \cdot \\ \cdot \vec{\mathrm{R}}(\hat{\mathbf{l}}' \times \hat{\mathbf{N}}', \hat{\mathbf{l}}' \times \hat{\mathbf{l}})F(\mathbf{r},\mathbf{r}')\Theta(\mathbf{r},\mathbf{r}'), \tag{20}$$

where $\mathbf{Q}_n$ is the vector weight of the beam with its components corresponding to the components of the Stokes vector, $M$ is the mean operator.

The expression (19) is called the local estimate of the Monte Carlo method and allows illuminance to estimate at a point of interest $\mathbf{r}$ of space in which multiple reflections are being modelled.

## 4. IMPLEMENTATION OF THE MATHEMATICAL MODEL AND THE RESULTS

The method of solution of GIE with consideration of polarisation by means of local estimates of the Monte Carlo method proposed above was implemented in MATLAB. A "room" with dimensions of 1×1×1 was selected as a scene for modelling multiple reflections; in the middle of its "ceiling", a disc-shaped lambertian source with radius of 0.05 is installed. On the "floor" of the room, at which nine points illuminance is estimated, are located (Fig. 2).

As the reflection matrix, the sum of two matrices was used:

$$\vec{\rho} = a\vec{\rho}_{ô} + (1-a)\vec{\rho}_{ë}, \tag{21}$$

where $\vec{\rho}_{ô}$ is the Mueller matrix (for Fresnel reflection in the case under consideration); $\vec{\rho}_{ë}$ is the matrix of lambertian reflection; $a$ is the fraction of Fresnel reflection $(0 < a < 1)$. The matrix $\vec{\rho}_{ë}$ is the zero matrix with the only non-zero $\rho_{ë,11}$ element, which is equal to the reflection coefficient.

Let us expand on the numerical algorithm which was used in the research programme for modelling interaction of the beam and the surface. Practical implementation of the above approach appears to be a non-trivial problem due to appearing of the δ-function in the indicatrix expression in the case of Fresnel reflection. Its availability gives rise to impossibility of playout of a new direction of a beam after interaction with the Fresnel surface and thus negates efficiency of local estimates.

To bypass this problem, the following algorithm is used. After the beam gets into the surface, the random parameter $\alpha$, uniformly distributed over the interval $(0, 1)$. is played out. Then, if $\alpha < a$, the new direction of the beam is calculated using formula

(6); otherwise, the beam is played out in accordance with the diffusion law. In accordance with the selected variant of interaction, Fresnel or lambertial reflection matrix is used.

The following parameters of surface were set as input variables: the reflection coefficient is 0.5, the refraction coefficient is 1.5, the parameters *a* varied from 0 to 1. The graphs in Fig. 3 demonstrate distributions of illuminance on the surface of the "floor" obtained both with and without consideration of polarisation with different values of the parameter *a*.

## 5. CONCLUSION

The above results show that consideration of polarisation significantly affects the values of illuminating parameters obtained while calculation. In case of $a = 0,6$, the difference between values exceeds 20 %, whereas in the limiting case it exceeds 30 %. Therefore, consideration of polarisation is required for solution of a number of lighting engineering problems related to modelling light distribution.

It is necessary to note that the above described mathematical model of multiple reflections of light with consideration of polarisation stays within the framework of standard photometric terms but is generalised for the case of polarisation. Significant difference from the standard model is that luminance is transformed from scalar value into vector value and the reflection coefficient becomes a matrix. Also it becomes necessary to take rotation of the reference plane after each interaction of light with substance into account.

As a result of the work, the global illumination equation was obtained for the polarisation case. This allows us to use the same methods which will consider the state of light polarisation after a certain modification. For instance, the expression for local estimate of MCM with consideration of polarisation was obtained. This method appears to be the most promising one nowadays since it allows simultaneous estimation at all points of interest of the scene based on just one beam and accelerates calculation averagely by (80–90) times as compared to direct modelling methods.

The following stages of research in this area may involve development of a more detailed model of light diffusion on surfaces of the studied scene and in the under-surface layer of the medium as well as its experimental verification.

## REFERENCES

1. Mishchenko M.I., Lacis A.A., Travis L.D. Errors induces be the neglect of polarization in radiance calculations for Rayleigh-scattering atmospheres // Journal of Quantitative Spectroscopy and Radiative Transfer, 1994, T. 51, № 3, pp. 491–510.

2. Wolff L.B., Kurlander D.J. Ray tracing with polarization parameter // IEEE Computer Graphics and Applications, 1990, T. 10, № 6, pp. 44–55.

3. Wilkie A., Tobler R., Ulbricht C. et al. An analytical model for skylight polarization // Proceedings EGSR, 2004, pp. 387–399.

4. Ghosh A., Chen T., Peers P. et al. Circularly polarized spherical illumination reflectometry // ACM Transaction on Graphics, 2010, T. 29, № 6, pp. 1–12.

5. Mojzik M., Skrivan T., Wilkie A., Krivanek J. Bi-Directional Polarised Light Transport // Eurographics Symposium on Rendering. 2016.

6. Zhdanov D., Ershov S., Shapiro L., Sokolov V., Voloboy A. et al. Realistic image synthesis in presence of birefrigent media by backward ray tracing technique // Proc. SPIE10694, Computational Optics II, 106940D (28 May 2018)

7. Kozlov D.S., Debelov V.A. Algorithm for Photorealistic Rendering of Transparent Optically Active Isotropic Crystals // Programming and Computer Software, 2015, T. 41, № 5, pp. 267–272.

8. Rosenberg G.V. Light Beam [Luch sveta] // Advances of Physics [Uspekhi fizicheskikh nauk], 1977, Vol. 121, Issue 1, pp. 97–138.

9. Apresyan L.A., Kravtsov Yu.A. The Theory of Radiation Transmission: Statistical and Wave Aspects [Teoriya perenosa izlucheniya: statisticheskiye i volnovyye aspekty] // Nauka. Chief editorial board of literature on physics and mathematics, Moscow, 1983, 216 p.

10. Stokes G.G. On the composition and resolution of streams of polarized light from different sources // Transactions of the Cambridge Philosophical Society. 1852, T. IX. p. 399.

11. Rosenberg G.V. Stokes Parameter Vector [Vektor-parametr Stoksa] // Advances of Physics [Uspekhi fizicheskikh nauk], 1955, Vol. LVI, Issue I, pp. 77–110.

12. Shurcliff W. Polarised Light [Polyarizovanniy svet] // Mir, Moscow, 1965, 264 p.

13. Prigarin S.M. The Basics of Statistical Modelling of Polarised Optical Radiation Transmission [Osnovy statisticheskogo modelirovaniya perenosa polyarizovannogo opticheskogo izlucheniya] // Study Guide, Novosibirsk State University, Novosibirsk, 2010, 109 p.

14. Budak V.P. Visualisation of Radiance Distribution in Three-Dimensional Scenes [Vizualizatsiya raspredeleniya yarkosti v tryokhmernykh stsenakh nablyudeniya] // MEI, Moscow, 2000, 136 p.

15. Kajiya J.T. The rendering equation // In Proceedings of SIGGRAPH, 1986, T. 20, № 4, pp. 143–150.

16. Kalos M.H. On the Estimation of Flux at a Point by Monte Carlo // Nuclear Science and Engineering, 1963, T. 16, № 1, pp. 111–117.

17. Budak V.P., Zheltov V.S., Kalatutskiy T.K. Monte Carlo Method Local Estimates in Solution of the Global Illumination Equation with Consideration of Spectral Representation of Objects [Lokalnyye otsenki metoda Monte Carlo v reshenii uravneniya globalnogo osveshcheniya s uchyotom spektralnogo predstavleniya obektov] // Computer Studies and Modelling [Komputernyye issledovaniya i modelirovaniye], 2012, Vol. 4, Issue 1, pp. 75–84.

***Vladimir P. Budak***,
Professor, Doctor of Technical Sciences. In 1981, he graduated from the Moscow Power Engineering Institute (MPEI). At present, he is the Editor-in-chief of the Svetotekhnika / Light & Engineering journal, Professor of the Light Engineering sub-department of NRU MPEI. Corresponding member of the Academy of Electrotechnical Sciences of Russia

***Anton V. Grimaylo,***
Master Student of NRU MPEI (Light Engineering sub-department)