Light & Engineering
Vol. 28, No. 6, pp. 118–126, 2020

https://doi.org/10.33383/2020-041
Svetotehknika # 5, 2020, pp. 44–51

DETERMINATION OF EFFICIENT MODES OF OPTICAL RADIATION EXPOSURE FOR CONTROL OF HUMAN CIRCADIAN ACTIVITY

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ABSTRACT

The article describes the stages of transformation of the direct and diffused components of solar radiation in the short-wave and long-wave spectral channels of the human circadian activity control path. The method of determination of the dependences of irradiance on Solar altitude angle as well as the values of radiant exposure in the circadian region of the optical spectre required for efficient control of human circadian activity. An example of utilisation of the developed method is provided. Correspondence between the results of calculations based on the proposed method and the results of independent experimental studies is demonstrated. The developed method allows us to formulate major light-engineering requirements to characteristics of emitting installations controlling human circadian activity, preventing and eliminating its deregulations.

Keywords: circadian activity, Solar altitude angle, components of solar radiation, thermodynamic temperature, spectral channels, effective irradiance, radiant exposure, effect modes, calculation method

1. INTRODUCTION

Daily changes of the characteristics of optical radiation of the Sun reaching the surface of the Earth are the main physical factor affecting human circadian activity (CA). These changes of characteristics of solar radiation (SR) are caused by current position of the Earth in the course of its orbital movement around the Sun as well as its current altitude angle common for a specific calculation point of the Earth surface.

When conducting light-engineering and biological studies, it is necessary to have information on the dependence of effective[1] irradiance E affecting CA of human body on Solar altitude angle h at a particular point of the Earth surface. This information as well as the values of radiant exposure H at different calculation points of the Earth surface is also necessary for design of specialised emitting installations for prevention and elimination of circadian deregulations.

Prevention and elimination of circadian deregulations allows us to save psychosomatic health and maintain necessary level of general and visual performance of human and provides maintenance of daily intellectual activity.

This work aims at development of the method of determination of effective energy characteristics of Solar and artificial radiations affecting human body CA.

2. INPUT DATA

Spectral density of radiant exitance of the Sun photosphere $m_{\mathrm{eS}}(\lambda, T)$ described by the Planck function [1] is used as the model of Solar radiation:

$$m_{\mathrm{eS}}(\lambda,T) = C_1\lambda^{-5}\left(\exp\frac{C_2}{\lambda T}-1\right)^{-1},$$

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[1] Hereinafter "effective" means the value of human body reaction to the effect of irradiance and spectral irradiance formed by SR in the circadian region of the spectre as well as to radiant exposure within the range of Solar altitude angles corresponding to CA control.

where λ is the radiation wavelength, T is the thermodynamic temperature of blackbody radiation, $C_1 \approx$ $\approx 3.742\cdot10^{-16}$ W·m^2 and $C_2 \approx 1.439\cdot10^{-2}$ m·K [2].

Spectral irradiance (SI) formed by SR normally incident on a site located at the upper border of the Earth atmosphere is expressed as [3]:

$$e_{\text{eS}}(\lambda,T)=\left(\frac{r}{R}\right)^2 m_{\text{eS}}(\lambda,T), \qquad (1)$$

where T is the average thermodynamic temperature of SR with respect to the Sun photosphere, $r = 6.96\cdot10^5$ km is the Sun equatorial radius, $R = 1.496\cdot10^{12}$ km is the radius of circular orbit of the Earth [4, 5].

The value T depends significantly on the year number n within the Schwabe's 11-year cycle of Solar activity [6] and is expressed as [3]

$$T(n)=T_{\text{aver}}\left[1+0{,}027\sin\left(\frac{2\pi n}{11}-\frac{\pi}{2}\right)\right],$$

where $T_{\text{aver}}=0.5\cdot(T_{\min}+T_{\max})$, $T_{\min} \approx 5480$ K, $T_{\max} \approx 5780$ K are the values of thermodynamic temperature of SR at the upper border of the atmosphere corresponding to the minimum and maximum Solar activity, $0 \le n \le 11$.

As a result of SR distribution over the atmosphere, there are two components of the radiation formed at the Earth surface: the direct (*Dir*) component and the diffused (*Diff*) component. The values of thermodynamic temperature of the direct and diffused components of SR during daytime at a design point of the Earth surface depend on the Solar altitude angle, i.e. $T_{\text{Dir}} = T_{\text{Dir}}(h, n)$ and $T_{\text{Diff}} = T_{\text{Diff}}(h, n)$.

The functions $T_{\text{Dir}}(h, n)$ and $T_{\text{Diff}}(h, n)$ may be written as approximate dependences.

$$T_{\text{Dir(Diff)}}(h,n) \approx T_{\text{Dir(Diff)}}(h)+T(n). \qquad (2)$$

Analysis of literature with respect to experimental data on the dependences $T_{\text{Dir}}(h, n)$ and $T_{\text{Diff}}(h, n)$, in particular, [7], etc. as well as approximation of this data demonstrated that $T_{\text{Dir(Diff)}}(h)$ in (2) may be written as

$$T_{\text{Dir(Diff)}}(h)=a_{\text{Dir(Diff)}}\exp\left(-b_{\text{Dir(Diff)}}h\right). \qquad (3)$$

The values of spectral radiance of the direct and diffused components at the Earth surface formed by SR are written as

$$e_{\text{eS,Dir(Diff)}}\left[\lambda,T_{\text{Dir(Diff)}}(h,n)\right]=$$
$$=\left(\frac{r}{R}\right)^2\tau_{\text{Dir(Diff)}}(h)C_1\lambda^{-5}\left(\exp\frac{C_2}{\lambda T_{\text{Dir(Diff)}}(h,n)}-1\right)^{-1}, \qquad (4)$$

where $T_{\text{Dir(Diff)}}(h, n)$ are the dependences (2), $\tau_{Dir(Diff)}(h)$ are the dependences of integral transmittance factors of the direct and diffused components of SR in the spectral window of the atmosphere $300 \le \lambda \le 1200$ nm [8] with different atmosphere cloudiness.

The dependence $\tau_{Dir(Diff)}(h)$ in (4) has the form [3]

$$\tau_{\text{Dir(Diff)}}(h)=$$
$$=\tau_{\text{Dir(Diff)}}(90^\circ)0{,}5\left[1+\sin\left(d_{\text{Dir(Diff)}}h-g_{\text{Dir(Diff)}}\right)\right], \qquad (5)$$

where $\tau_{\text{Dir(Diff)}}(90^\circ)$ are the integral transmissivity factors of normal SR incidence on the Earth surface (maximum Solar altitude angle $h = 90^\circ$ is seen at the equator at noon of the vernal or autumnal equinox days), the factors $d_{\text{Dir(Diff)}}$, $g_{\text{Dir(Diff)}}$ for different cloudiness of atmosphere are given in [9].

SI of the direct and the diffused components of SR on the Earth surface as per the relation (4) are received by retinal detectors characterised by the function of relative spectral circadian efficiency (FRSCE) in the following form [10]

$$c(\lambda)=c_1(\lambda)+c_2(\lambda)=\frac{\alpha_1}{\sigma_1\sqrt{2\pi}}\exp\left[-\frac{(\lambda-\lambda_{1\max})^2}{2\sigma_1^2}\right]+$$
$$\frac{\alpha_2}{\sigma_2\sqrt{2\pi}}\exp\left[-\frac{(\lambda-\lambda_{2\max})^2}{2\sigma_2^2}\right], \qquad (6)$$

obtained after approximating the results of independent experimental studies by G.K. Brainard and K. Thapan and their contributors, in particular, [11–14].

The graphs of the function (6) and its components $c_1(\lambda)$ and $c_2(\lambda)$ are shown in Fig. 1. The differences between FRSCE obtained by Brainard and Thapan are just in relation between the maximums of the functions $c_1(\lambda)$ and $c_2(\lambda)$.

3. RESULTS

The functions $c_1(\lambda)$ and $c_2(\lambda)$ form the short-wave (*SW*) and the long-wave (*LW*) spectral channels of the *CA* control path [10] and conduct spectral

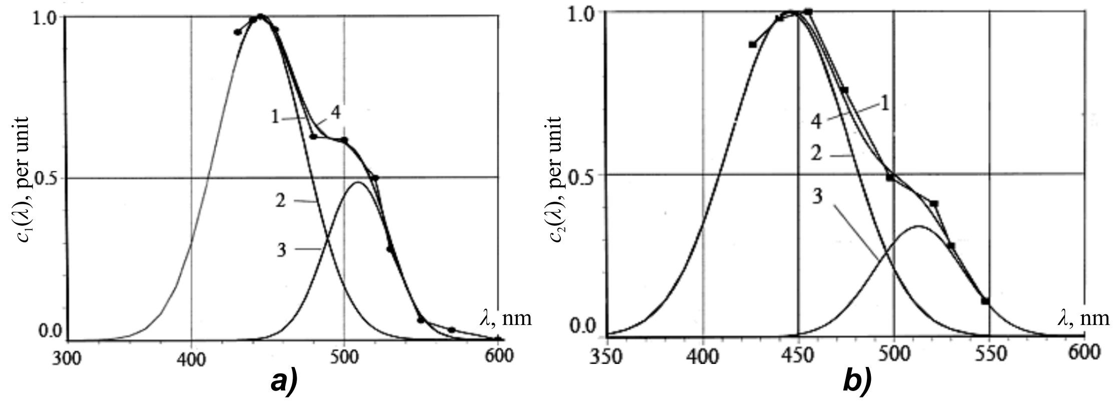

Fig. 1. FRSCEs obtained by Brainard (a) and Thapan (b), functions $c_1(\lambda)$, $c_2(\lambda)$ in the relation (9) and the sum $c_1(\lambda) + c_2(\lambda)$: *1* – results of the experimental studies [11–14], *2* – the function $c_1(\lambda)$, *3* – the function $c_2(\lambda)$, *4* – the function $c_1(\lambda) + c_2(\lambda)$ approximating the results of the studies [11–14]

and selective transformations of SR SI (4) described by the multiplications

$$e_{\mathrm{eS,Dir(Diff),SW(LW)}}\left[\lambda, T_{\mathrm{Dir(Diff)}}(h,n)\right]c_{1(2)}(\lambda) = \left(\frac{r}{R}\right)^2 \tau_{\mathrm{Dir(Diff)}}(h) C_1 \lambda^{-5}\left(\exp\frac{C_2}{\lambda T_{\mathrm{Dir(Diff)}}(h,n)} - 1\right)^{-1} \times c_{1(2)}(\lambda) \quad (7)$$

with isolation of the $350 \le \lambda \le 540$ nm spectral region from the broadband spectra (4) with spectral selection by only blue-sensitive (*B*) retinal cones, isolation of the $450 \le \lambda \le 570$ nm spectral region with spectral selection by only retinal rods, and isolation of the $350 \le \lambda \le 570$ nm spectral region with cooperative spectral selection by the *B* cones and rods of the retina. Obviously, after spectral and selective processing of the function (4) by the functions $c_1(\lambda)$, $c_2(\lambda)$, $c_1(\lambda) + c_2(\lambda)$ using the relation (7), the shape of the obtained spectrum is defined only by non-variable spectral characteristics of these functions which do not depend on Solar altitude angle.

Modification of $T_{\mathrm{Dir(Diff)}}(h, n)$ on the Earth surface with changes of Solar altitude angle during daytime leads not only to changes in chromaticity of SR (4) but also to λ-related displacement of the spectral maximum (4) based on the Wien's displacement law: $\lambda_{\max} \cdot T(h, n) = C_3 \approx 0.2898 \cdot 10^{-2}$ m·K [2].

Changes of values of $\lambda_{\max}$ with respect to the λ-fixed position of the functions $c_1(\lambda)$, $c_2(\lambda)$, $c_1(\lambda) + c_2(\lambda)$ and unchanged spectral form (7) leads to dependence of the energy characteristics (7) on Solar altitude angle.

Determination of the dependence of energy characteristics (7) on Solar altitude angle is based on two-step transformation of the arguments in (7). At the first step, we shift from the λ scale to the $\lambda_{\max}$ scale by means of linear (identity) transformation of the argument λ into the argument $\lambda_{\max}$. At the second step, $\lambda_{\max}$ is non-linearly functionally transformed into Solar altitude angle *h* using the Wien's displacement law by means of an inverse function of the function (2).

Functional transformation of the arguments in (7) is conducted using the known rule, e.g. [15, 16]:

$$S(y) = S(x)\left|\frac{dx}{dy}\right| = S[\varphi(y)]\left|\frac{d\varphi(y)}{dy}\right|, \quad (8)$$

where *x* is the basic argument, *y* is the new argument, $x = \varphi(y)$ is the inverse function of the original function $y = f(x)$.

At the first step of transformations of the argument λ into $\lambda_{\max}$, the argument $\lambda_{\max}$ directly substitutes λ in (4):

$$e_{\mathrm{Dir(Diff),SW(LW)}}\left(\lambda_{\mathrm{max,Dir(Diff)}} h, n\right) \cdot c_{1(2)}\left(\lambda_{\mathrm{max,Dir(Diff)}}\right) = \left(\frac{r}{R}\right)^2 \tau_{\mathrm{Dir(Diff)}}(h,n) C_1 \lambda^{-5}_{\mathrm{max,Dir(Diff)}} \times \left(\exp\frac{C_2}{\lambda_{\mathrm{max,Dir(Diff)}} T_{\mathrm{Dir(Diff)}}(h,n)} - 1\right)^{-1} \times \left|\frac{d\lambda}{d\lambda_{\mathrm{max,Dir(Diff)}}}\right| c_{1(2)}\left(\lambda_{\mathrm{max,Dir(Diff)}}\right). \quad (9)$$

In the relation (9), the modulus of the derivative is $\left|\dfrac{d\lambda}{d\lambda_{\max,\mathrm{Dir(Diff)}}}\right| = 1$ and the (9) itself describes the spectral form remaining homothetic with all values of h and n since this form is defined only by the functions $c_1(\lambda)$, $c_2(\lambda)$ or $c_1(\lambda) + c_2(\lambda)$.

The energy characteristics of the dependence of the function

$$e_{\mathrm{Dir(Diff),SW(LW)}}\left(\lambda_{\max,\mathrm{Dir(Diff)}} h, n\right) \cdot c_{1(2)}\left(\lambda_{\max,\mathrm{Dir(Diff)}}\right)$$

on the Solar altitude angle are defined using the Wien's displacement law.

With non-linear functional transformation, the Wien's displacement law $\lambda_{\max} \rightarrow h$ is written as

$$\lambda_{\max,\mathrm{Dir(Diff)}}(h,n) = C_3\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^{-1}. \quad (10)$$

In the non-linear transformation of the arguments $\lambda_{\max} \rightarrow h$ in the relation (9) using the rule (8), multipliers $\left|\dfrac{d\lambda_{\max,\mathrm{Dir(Diff)}}}{dh}\right| c_{1(2)}(h,n)$ are inserted instead of multipliers $\left|\dfrac{d\lambda}{d\lambda_{\max,\mathrm{Dir(Diff)}}}\right| c_{1(2)}\left(\lambda_{\max,\mathrm{Dir(Diff)}}\right)$.

The relation (9) is simply applied to the other multipliers.

The derivatives $\left|\dfrac{d\lambda_{\max,\mathrm{Dir(Diff)}}}{dh}\right|$ are written as

$$\left|\frac{d\lambda_{\max,\mathrm{Dir(Diff)}}}{dh}\right| = C_3\left|\frac{d\left[T_{\mathrm{Dir(Diff)}}(h)\right]}{dh} \cdot \frac{1}{\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^2}\right|. \quad (11)$$

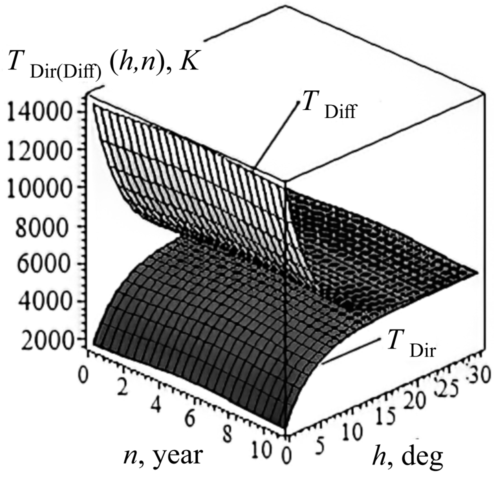

Fig. 2. The graphs of the functions $T_{\mathrm{Dir}}(h, n))$ and $T_{\mathrm{Diff}}(h, n)$ at a random design point of the Earth surface in the 11-year cycle of solar activity

After applying (10) and (11) to the relation (9), it describes just the dependence of its form on the values h and n per unit:

$$e_{\mathrm{Dir(Diff),SW(LW)}}\left\{C_3\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^{-1}\right\}\Big|_{\mathrm{rel}} =$$

$$= \left(\frac{r}{R}\right)^2 \tau_{\mathrm{Dir(Diff)}}(h,n) C_1 C_3^{-5}\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^5 \times$$

$$\times\left(\exp\frac{C_2}{\left\{C_3\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^{-1}\right\}} - 1\right)^{-1} \times$$

$$\times C_3\left|\frac{d\left[T_{\mathrm{Dir(Diff)}}(h)\right]}{dh} \frac{1}{\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^2}\right| c_{1(2)}(h,n). \quad (12)$$

Simplification of (12) transforms it into

$$e_{\mathrm{Dir(Diff),SW(LW)}}(h,n)\big|_{\mathrm{rel}} =$$

$$= e_{\mathrm{Dir\,eS}}(h,n)\left|\frac{dT_{\mathrm{Dir(Diff)}}(h,n)}{dh}\right| c_1(h,n)\big|_{\mathrm{rel}} =$$

$$= \left(\frac{r}{R}\right)^2 \tau_{\mathrm{Dir(Diff)}}(h,n) C_1 C_3^{-5} \times$$

$$\times\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^5\left(\exp\frac{C_2}{C_3} - 1\right)^{-1} \times$$

$$\times C_3\left|\frac{d\left[T_{\mathrm{Dir(Diff)}}(h)\right]}{dh} \frac{1}{\left[T_{\mathrm{Dir(Diff)}}(h) + T(n)\right]^2}\right| c_{1(2)}(h,n). \quad (13)$$

For illustrative purposes, the products $C_3C_3^{-5}$, $\left[T_{\mathrm{Dir(Diff)}}(h,n)\right]^5$ and $\dfrac{1}{\left[T_{\mathrm{Dir(Diff)}}(h,n)\right]^2}$ in (12) and (13) are not simplified.

The curve (13) is nearly bell-shaped, with its maximums at different combinations of the direct and diffused components of SR and *SW* and *LW* spectral channels, i.e. at $h_{\max,\,\mathrm{Dir(Diff),\,SW\,(LW)}}$.

Introduction of the product $c_{1(2)}(\lambda)$ from (9) in (13) as an additional multiplier allows us to obtain the dependences of SI (per unit) on two variables (h and λ) in the form of a three-dimensional control signal $e_{SW(LW)}(\lambda,h,n)|_{\mathrm{rel}}$ at input of the *suprachiasmatic nucleus of hypothalamus* (SCN):

$$e_{\mathrm{Dir(Diff),SW(LW)}}(\lambda,h,n)\big|_{\mathrm{rel}} = \left(\frac{r}{R}\right)^2 \tau_{\mathrm{Dir(Diff)}}(h,n)C_1C_3^{-5}\left[T_{\mathrm{Dir(Diff)}}(h)+T(n)\right]^5 \times \times\left(\exp\frac{C_2}{C_3}-1\right)^{-1}\left|-\frac{dT_{\mathrm{Dir(Diff)}}(h,n)}{dh}\right| \times \times c_{1(2)}(h,n)c_{1(2)}(\lambda). \qquad (14)$$

Like that of the function (13), the graph of (14) per unit is also bell-shaped.

Dependences of absolute effective values of (13) on current values of Solar altitude angle with each possible combination of the direct and diffused components of SR with the functions $c_{1(2)}(h,\ n)$ are defined using the calculated values of $h = h_{\mathrm{max,\ Dir(Diff),\ SW\ (LW)}}$ corresponding to the maximum values of (13). Then the relation (13) is normalised by dividing its left and right sides by the values of (13) calculated at $h = h_{\mathrm{max,\ Dir(Diff),\ SW\ (LW)}}$, i.e. the following functions are defined

$$e_{\mathrm{norm,Dir(Diff),SW(LW)}}(h,n) = = \frac{e_{\mathrm{Dir(Diff),SW(LW)}}\left\{C_3\left[T_{\mathrm{Dir(Diff)}}(h)+T(n)\right]^{-1}\right\}\big|_{\mathrm{rel}}}{e_{\mathrm{Dir(Diff),SW(LW)}}\left\{C_3\left[T_{\mathrm{Dir(Diff)}}\left(h=h_{\mathrm{max,\ Dir(Diff),SW(LW)}}\right)+ +T(n)\right]^{-1}\right\}\big|_{\mathrm{rel}}}. \qquad (15)$$

Using the relation (7), the dependences of the absolute values of SI

$$e_{\mathrm{eS,Dir(Diff),SW(LW)}}\times \times\left[\lambda,T_{\mathrm{Dir(Diff)}}\left(n,h=h_{\mathrm{max,Dir(Diff),SW(LW)}}\right)\right]c_{1(2)}(\lambda)$$

on λ are defined for the calculated values of Solar altitude angle $h = h_{\mathrm{max,\ Dir(Diff),\ SW(LW)}}$ corresponding to the maximum values of (13).

The dependences of absolute values of SI at input of SCN (3D-represented) are obtained using the relation (7) by multiplying

$$e_{\mathrm{eS,Dir(Diff),SW(LW)}}\times \times\left[\lambda,T_{\mathrm{Dir(Diff)}}\left(n,h=h_{\mathrm{max,Dir(Diff),SW(LW)}}\right)\right]c_{1(2)}(\lambda)$$

by the normalised functions (15).

Maximum absolute values of SI at output of SCN (depending just on the values of h) are obtained using (7) from the product

$$e_{\mathrm{Dir(Diff),SW(LW)}}(\lambda,h) = e_{\mathrm{norm,Dir(Diff),SW(LW)}}(h,n)\times e_{\mathrm{eS,Dir(Diff),SW(LW)}}\times \times\left[\lambda,T_{\mathrm{Dir(Diff)}}\left(n,h_{\mathrm{max,Dir(Diff),SW(LW)}}\right)\right]c_{1(2)}(\lambda)$$

by inserting the values of λ equal to 445 nm and 505 nm corresponding to the maximum values of the functions $c_1(\lambda)$ and $c_2(\lambda)$. The dependences $e_{,\ \mathrm{Dir(Diff),\ SW(LW)}}(h,\ n)$ at SCN output obtained after the said substitution are the signals directly controlling daily activity of epiphysis, which releases melatonin to blood plasma. Daily variations of melatonin concentrations in blood plasma ultimately lead to daily control of human nonconscious biological reactions including CA. CA directly affects different characteristics of visual perception, which form the basis of static and dynamic building of the mental world model and uses this basis for control of human higher intellectual activity including concrete and abstract thinking.

The developed method and the product of the relations (7) and the normalised functions (15) allows us to obtain the dependences of irradiance within the circadian region of spectrum on Solar altitude angle ($E(h)$) which are important for experimental and theoretical studies of CA, as well as the values of radiant exposure H within the range $\Delta h = h_{\mathrm{fin}} - h_{\mathrm{init}}$ of modification of the function $E(h)$, where h_{fin} and h_{init} are the initial and final values of Solar altitude angle respectively.

Irradiance $E(h)$ is written as

$$E(h) = e_{\mathrm{norm,Dir(Diff),SW(LW)}}(h,n)\times \times\int_{\lambda_{\mathrm{init}}}^{\lambda_{\mathrm{fin}}} e_{\mathrm{eS,Dir(Diff),SW(LW)}}\left[\lambda,T_{\mathrm{Dir(Diff)}}\left(h_{\mathrm{max,Dir(Diff),SW(LW)}},n\right)\right]\times \times c_{1(2)}(\lambda)d\lambda, \qquad (16)$$

and radiant exposure is written as

$$H(\Delta h) = \int_{\lambda_{\mathrm{init}}}^{\lambda_{\mathrm{fin}}}\int_{0^\circ}^{90^\circ} e_{\mathrm{eS,Dir(Diff),SW(LW)}} \left[\lambda,T_{\mathrm{Dir(Diff)}}\left(h_{\mathrm{max,Dir(Diff),SW(LW)}},n\right)\right] c_{1(2)}(\lambda)e_{\mathrm{norm,Dir(Diff),SW(LW)}}(h,n)d\lambda dh. \qquad (17)$$

In (16) and (17), λ_{init} and λ_{fin} are the values of λ limiting the considered spectral region defined by the functions $c_1(\lambda)$, $c_2(\lambda)$, $c_1(\lambda) + c_2(\lambda)$, 0° and 90° are the limits of the possible range of Solar altitude angle.

The example below contains the results of calculations using the developed method which describe processing of the direct and diffused components of SR in the *SW* spectral channel of the CA control path and define effective modes of the effect of radiation with maximum solar activity ($n = 5.5$), for instance, in conditions of clear sky.

In the conditions under consideration, the function $T_{\mathrm{h_{max},Dir(Diff),SW(LW)}}(h,n)$ (2) with (3) taken into account in the expression of spectral irradiance of the direct and diffused components (4) is written as

$$T_{\mathrm{Dir(Diff)}}(h,n) \approx a_{\mathrm{Dir(Diff)}} \exp\left(-b_{\mathrm{Dir(Diff)}}h\right) + T(n). \quad (18)$$

In (18), $a_{\mathrm{Dir}} = -3780$, $b_{\mathrm{Dir}} = 0.2444$, $a_{\mathrm{Dff}} = 8950$, $b_{\mathrm{Diff}} = 0.2084$, $T(n = 5.5) = 5780$.

The graphs of the dependence (18) are shown in Fig. 2.

In (5), the values of the coefficients are as follows: τ_{Dir} (90°) = 0.73, $d_{\mathrm{Dir}} = 0.0348$, $g_{\mathrm{Dir}} = 1.55$, τ_{Diff} (90°) = 0.13, $d_{\mathrm{Diff}} = 0.029$, $g_{\mathrm{Diff}} = 1.04$, and the values of the coefficients in (6), for instance, with data from [13] are as follows: $\alpha_1 = 72.56\times10^{-9}$ m, $\sigma_1 = 28.99\times10^{-9}$ m, $\lambda_{1,\max} = 445\cdot10^{-9}$ m, $\alpha_2 = 25.89\times10^{-9}$ m, $\sigma_2 = 21.21\cdot10^{-9}$ m, $\lambda_{2,\max} = 505\times10^{-9}$ m.

In accordance with the Wien's displacement law, the expressions for $\lambda_{\max,\mathrm{Dir}}$ and $\lambda_{\max,\mathrm{Diff}}$ (10), with the expression (4) taken into account, are written as

$$\lambda_{\max,\mathrm{Dir}} = C_3\left[-3780\exp(-0{,}2444h)+5780\right]^{-1}, \quad (19)$$

$$\lambda_{\max,\mathrm{Diff}} = C_3\left[8950\exp(-0{,}2084h)+5780\right]^{-1}. \quad (20)$$

The derivatives in (9) are as follows:

$$\left|\frac{d\lambda_{\max,\mathrm{Dir}}}{dh}\right| = \frac{2{,}6773\exp(-0{,}2444h)}{\left[-3780\exp(-0{,}2444h)+5780\right]^2}, \quad (21)$$

$$\left|\frac{d\lambda_{\max,\mathrm{Diff}}}{dh}\right| = \frac{5.4053\exp(-0{,}2084h)}{\left[8950\exp(-0{,}2084h)+5780\right]^2}. \quad (22)$$

After inserting the expression (6) with its coefficients α_1, σ_1, $\lambda_{1,\max}$, α_2, σ_2, $\lambda_{2,\max}$ and the expressions (19)–(22) into the relation (13) (for maximum values of the direct component of SR in the *SW* spectral channel), the expression $e(h,n)\big|_{\mathrm{rel}}$ at input of SCN in expanded form is written as

$$e_{\mathrm{Dir,SW}}(h,n)\big|_{\mathrm{rel}} =$$

$$= \left(\frac{r}{R}\right)^2 0{,}73\cdot0{,}5\left[1+\sin(0{,}0348h-1{,}55)\right]\times$$

$$\times C_1\left\{\frac{C_3}{\left[-3780\exp(-0{,}2444h)+5780\right]}\right\}^{-5}$$

$$\left[\exp\frac{C_2}{C_3}-1\right]^{-1}\times$$

$$\times\frac{2{,}6773\exp(-0{,}2444h)}{\left[-3780\exp(-0{,}2444h)+5780\right]^2}\frac{\alpha_1}{\sigma_1\sqrt{2\pi}}\times$$

$$\times\exp\left[-\frac{\left(\frac{C_3}{\left[-3780\exp(-0{,}2444h)+5780\right]}-445\cdot10^{-9}\right)^2}{2\sigma_1^2}\right]. \quad (23)$$

The similar expression for the diffused component of SR in the *SW* spectral channel is written as

$$e_{\mathrm{Diff,SW}}(h,n) =$$

$$= \left(\frac{r}{R}\right)^2 0{,}13\cdot0{,}5\left[1+\sin(0{,}029h-1{,}04)\right]\times$$

$$\times C_1\left\{\frac{C_3}{\left[8950\exp(-0{,}2084h)+5780\right]}\right\}^{-5}\times$$

$$\times\left[\exp\frac{C_2}{C_3}-1\right]^{-1}\times$$

$$\times\frac{5{,}4053\cdot\exp(-0{,}2084h)}{\left[8950\cdot\exp(-0{,}2084h)+5780\right]^2}\times$$

$$\times\left(\frac{\alpha_1}{\sigma_1\sqrt{2\pi}}\right)\times$$

$$\times\exp\left[-\frac{\left(\frac{C_3}{\left[8950\exp(-0{,}2084h)+5780\right]}-445\cdot10^{-9}\right)^2}{2\sigma_1^2}\right].$$

The dependence of SI on $\lambda_{\max,\mathrm{Dir(Diff)}}$ and h at input of SCN is written in the form

$$e_{\text{Dir(Diff)}}\left(\lambda_{\text{max,Dir(Diff)}},h,n\right)\Big|_{\text{rel}} =$$

$$=\left(\frac{r}{R}\right)^2 \tau_{\text{Dir(Diff)}}\left(h,n\right)C_1\lambda^{-5}_{\text{max,Dir(Diff)}}\times$$

$$\times\left(\exp\frac{C_2}{\lambda_{\text{max,Dir(Diff)}}T_{\text{Dir(Diff)}}\left(h,n\right)}-1\right)^{-1}\times$$

$$\times\, c_{1(2)}\left(\lambda_{\text{max,Dir(Diff)}}\right)\left|\frac{d\lambda_{\text{max,Dir(Diff)}}}{dh}\right|c_{1(2)}\left(h,n\right).$$

Solar altitude angle corresponding to the maximum value $e_{\text{Dir, SW}}(h)$, h_{max} calculated from the expression (23) equals to 16.14° and the corresponding maximum value of the function $e_{\text{Dir,SW}}\left(h\right)\big|_{\text{rel}}$ equals to 0.01533.

Integration of the function $e_{\text{eS, Dir, SW}}(\lambda, h)$ from the expression (7) over λ at $h_{\text{max}} = 16.14°$ gives the maximum value of irradiance:

$$E_{\text{Dir,SW}}\left(h_{\text{max}}=16{,}14^{\circ}\right)=$$

$$\int\limits_{350\cdot10^{-9}}^{570\cdot10^{-9}} e_{\text{Dir,SW}}\left(\lambda\right)\Big|_{h_{\text{max}}=16{,}14^{\circ}}\,d\lambda = 6{,}915\ \text{W}\cdot\text{m}^{-2}. \quad (24)$$

After multiplying the calculated value of the integral (24) by the normalised function $e_{\text{norm,Dir,SW}}\left(h,n\right)$ derived from $e_{\text{norm,Dir(Diff),SW(DW)}}\left(h,n\right)$ by means of the expression (15), the expression for the irradiance dependence $E_{\text{Dir, SW}}(h)$ formed by the direct component of SR in the *SW* spectral channel is written as

$$E_{\text{Dir,SW}}\left(h\right)=\frac{6{,}9145}{0{,}01533}e_{\text{Dir eS}}\left(h\right)\left|\frac{d\lambda_{\text{max,Dir}}}{dh}\right|c_1\left(h\right)=$$

$$=4{,}5115\cdot10^2\left(\frac{r}{R}\right)^2\times$$

$$\times\, C_1\left\{\frac{C_3}{\left[-3780\exp\left(-0{,}2444h\right)+5780\right]}\right\}^{-5}\times$$

$$\times\left[\exp\frac{C_2}{C_3}-1\right]^{-1}\cdot\ 0{,}73\times$$

$$\times\, 0{,}5\left[1+\sin\left(0{,}0348h-1{,}55\right)\right]\times$$

$$\times\frac{2{,}6773\exp\left(-0{,}2444h\right)}{\left[-3780\exp\left(-0{,}2444h\right)+5780\right]^2}\times$$

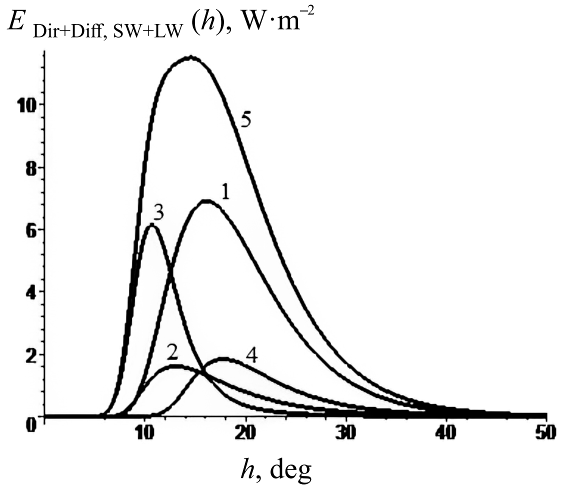

Fig. 3. The dependences of effective irradiance on Solar altitude angle under combined effect of direct and diffused components of SR simultaneously on the *SW* and *LW* spectral channels: *1* – irradiance under effect of the direct component of SR on the *SW* spectral channel, *2* – irradiance under effect of the direct component of SR on the *LW* spectral channel, 3 – irradiance under effect of the diffused component of SR on the *SW* spectral channel, *4* – irradiance under effect of the diffused component of SR on the *LW* spectral channel, 5 – irradiance corresponding to the sum of the direct and diffused components of SR simultenaously affecting the *SW* and *LW* spectral channels

$$\times\left(\frac{\alpha_1}{\sigma_1\sqrt{2\pi}}\right)\times$$

$$\times\exp\left[-\frac{\left(\frac{C_3}{\left[-3780\exp\left(-0{,}244h\right)+5780\right]}-445\cdot10^{-9}\right)^2}{2\sigma_1^2}\right].(25)$$

The graph of the dependence of absolute values of effective irradiance on Solar altitude angle as per the expression (25) is shown in Fig. 3.

The absolute value of effective radiant exposure $H_{\text{Dir, SW}}\ (\Delta h)$ within the interval of Solar altitude angle which corresponds to the effect of the direct irradiance component $E_{\text{Dir, SW}}\ (h)$ on the *SW* spectral channel of the CA control path is calculated as

$$H_{\text{Dir,SW}}\left(\Delta h\right)=\int\limits_{0^{\circ}}^{90^{\circ}}E_{\text{Dir,SW}}\left(h\right)dh = 91{,}218\ \text{W}\cdot\text{m}^{-2}\cdot\text{deg}.$$

With combined effect of the direct and diffused components of SR on the *SW* and *LW* spectral channels simultaneously, the dependence of absolute values of effective irradiance on Solar altitude angle $E_{\text{Dir+Diff, SW+LW}}(h)$ is described by the sum

Table. Effective Modes of the Effect of Solar Radiation in the Circadian Region of Spectrum for Control of Human Body Circadian Activity

Energy characteristics → Types of effect ↓	h_{init}	h_{fin}	$h_{\max}$	$E(h_{\max})$, Wm^{-2}	H, Wm^{-2} deg
Direct SR in the *SW* spectral channel	7°	43°	16.14°	6.915	91.218
Direct SR in the *LW* spectral channel	6°	43°	13.04°	1.618	18.768
Diffused SR in the *SW* spectral channel	4.5°	34°	10.69°	6.164	38.100
Diffused SR in the *LW* spectral channel	10°	43°	17.78°	1.848	21.731
Direct and diffused SR in the *SW* spectral channel	4.5°	43°	12.56°	9.262	129.341
Direct and diffused SR in the *LW* spectral channel	6°	43°	16.75°	2.987	40.499
Direct SR in the *SW* and *LW* spectral channels	7°	43°	15.62°	8.263	109.987
Diffused SR in the *SW* and *LW* spectral channels	4.5°	43°	10.69°	6.164	59.858
Direct and diffused SR in the *SW* and *LW* spectral channels	4.5°	43°	14.10°	11.492	169.845

$$E_{\text{Dir+Diff,SW+LW}}\left(h\right)=E_{\text{Dir,SW}}\left(h\right)+E_{\text{Dir,LW}}\left(h\right)+$$

$$E_{\text{Diff,SW}}\left(h\right)+E_{\text{Diff,LW}}\left(h\right)=$$

$$=\frac{6{,}9145}{0{,}01533}e_{\text{Dir eS}}\left(h\right)\left|\frac{d\lambda_{\text{max,Dir}}}{dh}\right|c_1\left(h\right)+$$

$$+\frac{1{,}6179}{0{,}09506}e_{\text{Dir eS}}\left(h\right)\left|\frac{d\lambda_{\text{max,Dir}}}{dh}\right|c_2\left(h\right)+$$

$$+\frac{6{,}1639}{0{,}9512}e_{\text{Diff eS}}\left(h\right)\left|\frac{d\lambda_{\text{max,Diff}}}{dh}\right|c_1\left(h\right)+$$

$$+\frac{1{,}8434}{0{,}07545}e_{\text{Diff eS}}\left(h\right)\left|\frac{d\lambda_{\text{max,Diff}}}{dh}\right|c_1\left(h\right), \qquad (26)$$

the members of which are defined (similar to the expression (25)) using the developed method.

The graphs of the members of (26) $E_{\text{Dir, SW}}(h)$, $E_{\text{Dir, LW}}(h)$, $E_{\text{Diff, SW}}(h)$, $E_{\text{Diff, LW}}(h)$ and of their sum are shown in Fig. 3.

The absolute value of effective radiant exposure $H_{\text{Dir+Diff, SW+LW}}(\Delta h)$ within the interval of Solar altitude angle $0° \leq h \leq 43°$ which corresponds to simultaneous effect of the direct and diffused components of effective irradiance on the *SW* and *LW* spectral channels is expressed as

$$H_{\text{Dir+Diff,SW+LW}}=H_{\text{Dir,SW}}+H_{\text{Dir,LW}}+H_{\text{Diff,SW}}+$$

$$+H_{\text{Diff,LW}}=169{,}845\ \text{W}\cdot\text{m}^{-2}\cdot\text{deg}. \qquad .$$

The Table summarises the values of energy characteristics of CA control signals with different combinations of effects of the direct and diffused components of SR on *SW* and *LW* spectral channels.

The last line of the table contains the values of irradiance and radiant exposure in the case of normal human health condition (without fatal failure of one of the CA control path spectral channels).

The fifth line of the table corresponds to fatal failure of the *LW* spectral channel and the sixth line of the table corresponds to fatal failure of the *SW* spectral channel.

It is worth noting that, according to the available experimental data, CA is controlled exceptionally under direct effect of SR on human body, i.e. within the period between sunrise and sunset. The data in the table completely complies with such provision: CA control commences (in the morning) and finishes (in the evening) at Solar altitude angle $h = 4.5°$.

It is also important that the daily value of superior culmination of the Sun h_{fin} approximately equal to 43° corresponds to non-availability of circadian deregulations in real cases of SR effect (lines 7–9 of the Table) and to cases of fatal failure of one of spectral channels (lines 5 and 6 of the Table), Fig. 3. At the same time it is seen from Fig. 3 and the Table that CA control ends at $h_{\text{fin}} \geq 43°$. Processing of the results of independent experimental studies has shown that the value $h_{\text{fin}} = 43.13°$ corresponds to complete non-availability of human circadian deregulations at arbitrary values of latitude and the day of a year [17]. Further increase of Solar altitude angle, e.g. with decrease of latitude on a random day of a year, does not lead to changes in CA [17].

Correspondence between the values of h_{fin} defined using the proposed method and obtained after processing of the results of independent experimental studies confirms practicability of this method.

Adequacy of the results of calculations obtained using the latter confirms that it may be used in theoretical and experimental studies of CA considering the effect of optical radiation on human body.

CONCLUSION

The values of effective irradiance and radiant exposure shown in Fig. 3 and the Table may be used in the course of experimental studies of nonconscious daily reactions of human body to levels of affecting irradiance and radiant exposure with different combinations of the direct and diffused components of optical radiation on the *SW* and *LW* spectral channels of the CA control path. Application of the proposed method allows us to define the values of irradiance and radiant exposure controlling human CA also at different cloudiness [9]. In particular, the results of this work may form a basis for research of circadian deregulations caused by trans-meridian flights and shift working in facilities with continuous production cycle. Moreover, the information obtained in the course of the work may be used for development of special emitting installations for prevention of circadian deregulations, correction of CA if they are available and for maintenance of proper human CA in conditions of daily CA deficiency, which is especially important for enhancement of protection of national security in sub-polar and polar regions and development of their economic use.

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