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CALCULATION OF LIGHT DISTRIBUTION OF A CONVENTIONALLY POINT LIGHT SOURCE IN AN ARBITRARILY ORIENTED COORDINATE SYSTEM

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ABSTRACT

The article reviews calculation of total light distribution of several light sources (LS), which are differently oriented in space with their locations conventionally[1] being the same. It is proposed that luminous intensity curves (photometric body) of LSs are described in *IESNA* format (or in the format of tables, which is basically the same). Two methods of solving the problem are proposed. The first one is related to preliminary trigonometric interpolation of luminous intensity curves for each LS performed by means of discrete Fourier transformation (DFT). The second one is based on piecewise-linear interpolation of this curves using Delaunay triangulation. Both methods may be implemented by means of popular mathematic software (such as *Wolfram Mathematica* or *Octave*) and their applicability is confirmed experimentally.

Keywords: luminous intensity angular distribution, total luminous intensity distribution, photometric data, trigonometric interpolation, discrete Fourier transformation, piecewise-linear interpolation, Delaunay triangulation, coordinate system rotation, coordinate transformation

1. INTRODUCTION

Recently, the interest in the idea of design of lighting devices (LD) with units or tens of light emitting diodes (LED) or LED modules with secondary optic devices which are differently oriented in space has become evident in scientific publications [1–4]. Such approach has two advantages. First, it allows us to create an LD with a photometric body (PB) of any complexity using secondary optics with simple geometry. Second, by providing capability of rotation of specific LEDs (LED modules) in the structure of a LED based LD, it is possible to optimise its luminous intensity distribution depending on lighting conditions.

However, over the last decade, the range of secondary optics for LED based outdoor lighting luminaires has significantly increased, therefore, the said approach to development of luminaires of this category, in point of fact, has lost its necessity. At the same time, it is our opinion that it is still necessary, for instance, for development of crossbar luminaires for railroad lighting and lighting of production premises as well as luminaires for architectural lighting of buildings and structures. Accordingly, the studies for calculation of total angular distribution of luminous intensity (spatial light distribution) of a system of differently oriented LSs with known PB (or, which is the same, luminous intensity curves) are still necessary.

Solution of the problem is significantly complicated by the fact that PBs of initial LSs are three-dimensional. Even one of the recent studies on mathematical modelling of LED modules [5] reviews the dependence of light distribution on one vectorial angle, i.e., in fact, a two-dimensional problem is solved.

[1] Conventionally concentrated at a large distance from a photoelectric receiver

Nowadays, there is a method developed by S.G. Ashurkov and A.A. Bartsev [1] which allows us to solve a three-dimensional problem provided the initial PBs of LSs are axially symmetric. This article proposes two ways of solving this problem but without the said limitation, i.e. with non-symmetric initial PBs of LSs.

2. STATEMENT OF THE PROBLEM OF CALCULATION OF TOTAL LIGHT DISTRIBUTION

It is known that PB of a point LS is a function $I(\vec{e})$ expressing dependence of the values of luminous intensity I in direction $\vec{e}$. The latter may be defined by two angles in one of the systems: (A,α), (B,β) or (C,γ) [6]. In terms of mathematics, PB is a surface in a spherical coordinate system where I acts as a radius and angular coordinates depend on selection of the photometric system.

Usually PB is found by measurement data of a goniophotometer output in *IESNA* format [7]: in point of fact, in the form of a table where the values of angular coordinates are given with a specific increment and the values of luminous intensity correspond with each pair of such coordinate values.

Let us designate angular coordinates in a given spherical system as Θ and Φ (Fig. 1), $\Theta\in[0°, 180°]$ and $\Phi\in[0°, 360°]$ and let us perform measurements of the first angle with increment of $\Delta\Theta$ and of the second angle with increment of $\Delta\Phi$. Introducing the following designations:

$$\Theta_k = k\Delta\Theta,\ \Phi_l = l\Delta\Phi, \quad (1)$$

where $k = 0–N_\Theta$, $l = 0–N_\Phi$, $N_\Theta = 180°/\Delta\Theta$ and $N_\Phi = 360°/\Delta\Phi$, we will get that the following values of luminous intensity are known

$$i_{kl} = I(\Theta_k, \Phi_l). \quad (2)$$

Then let us consider several LSs located at one point with PB of each one known and expressed by (1) and (2). All PBs are described in the same photometry system, for instance, (C,γ), with the role of γ angle is played by Θ and the role of Φ is played by C. (It is evident to consider that measurement increments $\Delta\Theta$ and $\Delta\Phi$ are common for all LSs though this consideration is not crucial.)

The LSs are differently oriented in space and light distribution of each of them is described in its

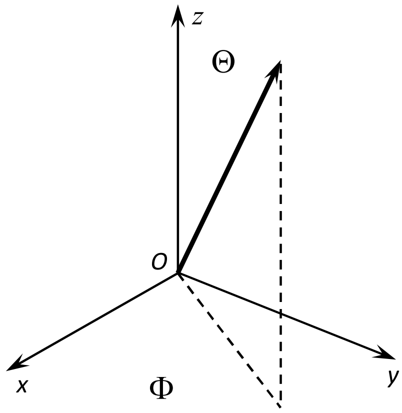

Fig. 1. Angular coordinates in a spherical system

own coordinate system rigidly bound to data of a specific LS. At the same time, relative positions of LSs are known, i.e. the sequence of rotations allowing own coordinate systems to combine is known.

The problem is as follows: total light distribution of the above described conventionally point LSs should be found.

Due to misalignment of their own coordinate systems, it is impossible to directly sum up the values of i_{kl} of different LSs in relevant nodes of the mesh. It is necessary to select some *common* coordinate system, to recalculate the $I(\vec{e})$ functions for each LS in it, and only then to sum up. To reduce calculations, the own coordinate system of one of the LSs may be selected as the common system or some other one may be selected.

Below, two methods of solving the stated problem are presented and the experimental set, by means of which the input data for verification of theoretical computations was performed and the methods were compared, is described.

3. USE OF TRIGONOMETRIC INTERPOLATION

Let us know the analytic expression for each LS:

$$I_j = I_j(\Theta,\Phi),\ j = 1–N, \quad (3)$$

where N is total number of LSs, the j index is used for their numbering, and the Θ and Φ angles correspond with the own coordinate system of the j-th LS.

As the transformation formulae combining own coordinate systems of different LSs are known, we may find the dependence between the angular coordinates in the own system and its coordinates Θ and φ in the common system (where PBs of LSs will be

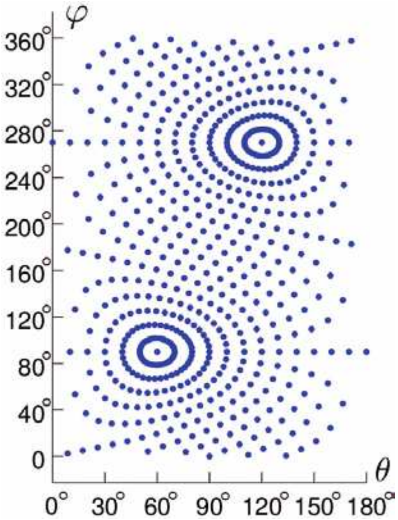

Fig. 2. Distortion of a regular mesh after 60° rotation around the *Oy* axis

summed up). The said dependence will have the following form for the *j*-th LS

$$\Theta = T_j(\theta,\varphi),\ \Phi = F_j(\theta,\varphi). \quad (4)$$

According to expressions (3) and (4), total light distribution of *N* LSs in the said common coordinate system will be expressed as

$$I(\theta,\varphi) = I_1(T_1(\theta,\varphi), F_1(\theta,\varphi)) + ... \\ ... + I_N(T_N(\theta,\varphi), F_N(\theta,\varphi)). \quad (5)$$

The said approach is detailed in [8] and in this work, model examples of its application are built.

To implement the said approach, it is necessary to recover the functions (3) by means of input data (1, 2).

Previous attempts to solve this problem are described in [9]. PB was expanded in degree of *cos θ.* Disadvantages of such solution are taking of only one angular variable into account (i.e. transition from space to plane) and low number of summands in the expanding (just 4). In their recent work [5], the authors increased accuracy of such expanding by multiplying the number of summands and advancing the methods of coefficient definition in such sums, however, the problem of transition from plane to space was not solved by them.

Possible approaches to solve the said problem different in terms of accuracy and complexity are described in [3, 10]. When interpolating photometric data, it is necessary to take PB (3) argument periodicity into account: it is obvious that Θ period of function I_j should be equal to 180° and that of Φ should be equal to 360°. Therefore, it is reasonable to calculate these functions in the form of double trigonometric series by Θ and Φ.

The data set in the form of equations (1,2) is finite, therefore, substantially the case in hand is trigonometric polynomial with $(N_\Theta + 1)\,N_\Phi$ summands; the coefficients of these polynomials are subject to definition:

$$I(\Theta,\Phi) = \sum_m \cos m\Theta \sum_n (a_{mn}\cos(n\Phi) + \\ + b_{mn}\sin(n\Phi)). \quad (6)$$

In real photometric experiments, increments $\Delta\Theta$ and $\Delta\Phi$ are small, therefore, the number of unknown variables is rather high: for instance, with $\Delta\Theta = 1°$ and $\Delta\Phi = 5°$, it is equal to 13,032. That is why this brings up the question of the fastest and the most accurate method of trigonometric interpolation.

As shown in [4], such method is the method of discrete Fourier transformation (DFT). In its simplest variant, it allows to roughly recover the periodic function by its known values. For array $\{x_k\}$ with period of *N*, DFT is defined by the formula

$$X_n = \sum_{k=0}^{N-1} x_k \exp(-i\frac{2\pi nk}{N});$$

then the continuous function

$$x(t) = \frac{1}{N}\sum_{n=0}^{N-1} X_n \exp\left(i\frac{2\pi nt}{N\Delta t}\right)$$

is periodic, and $\{x_k\}$ are its values obtained at values of *t* taken with increment of Δt [11].

The said transformation generalised for the periodic function by two its variables should be applied to the data set (2) after which it is necessary to define the real part in the obtained expression and then to reduce the number of summands excluding all expressions with coefficients a_{mn} and b_{mn} less than some previously defined value (it is defined with consideration of the desired accuracy) out of the sum.

So, according to the first algorithm of calculation of total light distribution of several LSs based on the results of photometric experiments, it is necessary to:

• Define light distribution (3) of each LS in its own coordinate system applying DFT to data (1)–(2);

• Define concretely the transformations (4) relating angular variables in own coordinate systems of LSs and in the common one to each other knowing relative positions of LSs;

• Calculate total luminous intensity distributions (common PB) in the common coordinate system using formula (5).

As a result of these actions, some formula (rather cumbersome) expressing the required function $I(\theta,\varphi)$ will be obtained. It may seem inconvenient but it may be used to compile the table of values in the form (2) and to obtain the expression in the form (6) in the common coordinate system by it.

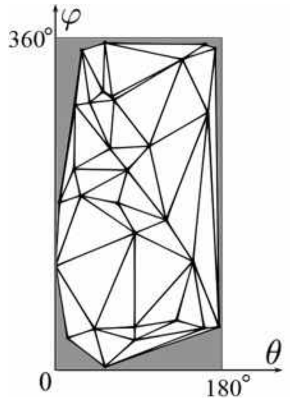

Fig. 3. Non-full covering of the region [0°; 180°]× × [0°; 360°] by the non-regular mesh

4. USE OF PIECEWISE-LINEAR INTERPOLATION

If PB (3) of each specific LS is not of interest, it is possible to try to avoid application of DFT. In this case, it is necessary to transfer to the common coordinate system from the beginning. The angular coordinates which the measured values of luminous intensity i_{kl} correspond with after this transition may be defined by the system (4) defined by θ and φ:

$$\theta = t_j(\Theta,\Phi),\ \varphi = f_j(\Theta,\Phi),\ j = 1\text{–}N. \tag{7}$$

After substituting the values of Θ_k and Φ_l for all N LSs into (2) and (7), the table of values of luminous intensity will be obtained in the following form

$$i_{kl} = I_j(\Theta_k,\Phi_l), \tag{8}$$

where, according to (7), $\theta_k = t_j(\Theta_k,\Phi_l)$ and $\varphi_l = f_j(\Theta_k,\Phi_l)$.

Since different functions t_j and f_j correspond with different LSs, after transformation of coordinates, the same angles Θ_k and Φ_l transform into different angles θ_k and φ_l. As a result, summing up of i_{kl} related to different LSs is impossible right after rotation. This problem did not arise using the analytic expressions (3) as they allow at any point to define I_j. Therefore, interpolation should be performed for all rotated LSs to define luminous intensity of different LSs at the same points.

$$\theta_p = p\Delta\Theta,\ \varphi_q = q\Delta\Phi, \tag{9}$$

where it is convenient to select corresponding increments of modification of angular variables in own coordinate systems and in the common one.

After rotation, the mesh Θ_k, Φ_l covering the interval [0°; 180°]×[0°; 360°] ceases to be regular (Fig. 2).

Among the methods applicable to such meshes, piecewise-linear interpolation is the simplest one as it provides acceptable accuracy with sufficient number of mesh nodes. That is why this method is used hereafter. For its implementation, it is necessary to perform triangulation of the region by the nodes of the obtained non-regular mesh, to define which of the triangles a specific point (9) gets into and to define I_j at this point knowing i_{kl} at apexes of the triangle.

Absolute error of such interpolation is known [3]: within each triangle, it does not exceed $M{\cdot}h^2/6$ where M is the largest value of second derivatives of the approximated function, h is the diameter of

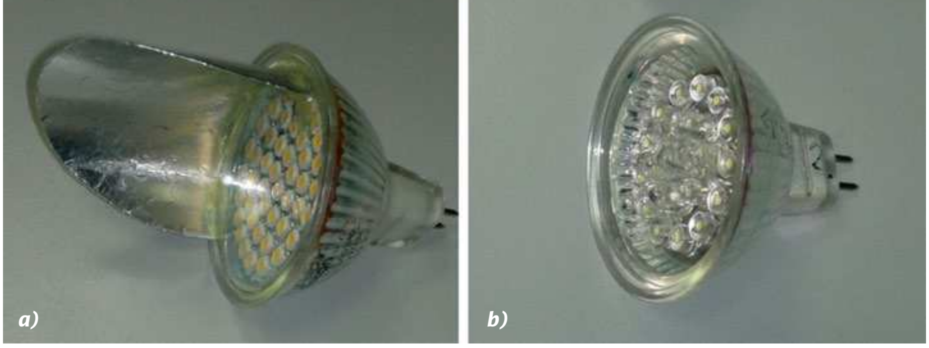

Fig. 4. Light sources used in the photometry experiment:
a – LS1; *b* – LS2

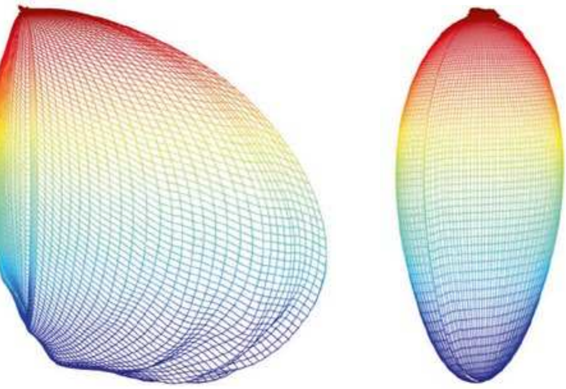

Fig. 5. Experimentally acquired photometric bodies of LS1 and LS2

the escribed circle of the triangle. To reduce the error, *h* should be minimised. For this purpose, fragmentation of the said region is performed by means of Delaunay triangulation for minimisation of the error [12]. With such approach, none of the mesh nodes gets into the escribed circle of any triangle. Delaunay triangulation is distinctive with minimal sum of radii of the escribed circles of all triangles. Therefore, it is this approach which "averagely" provides the least error.

The advantage of this algorithm is that it is supported by many mathematical software packages, e.g. *Mathematica* [13] or *Octave* and does not require additional programming.

Ultimately, the second algorithm of calculation of total light distribution consists of the following steps:

- Transfer to the common coordinate system using formulae (7) and (8);
- Fragmenting of the computational region by means of Delaunay triangulation;
- Piecewise-linear interpolation of photometric data of all LS's at the same points (9);
- Summing up of the values of luminous intensity of different LS's at points (9).

The result of implementation of this algorithm is the table of total values of luminous intensities.

Implementation of this plan is related to a number of complications.

First, in the own coordinate systems, the results of photometry at $\Theta = 0°$ provide different values of luminous intensity at different Φ angles although, based on introduction of the spherical coordinate system, these values should be equal (Fig. 1). The similar statement is correct at $\Theta = 180°$. This effect may be explained both by vibrations of the goniophotometer during measurements and by the fact that a luminaire may emit more or less light at different moments whereas photometric measurement is not performed in a moment.

When DFT is used, this phenomenon is not crucial because regularity of positions of points (Θ_k, Φ_l) is of key importance; with piecewise-linear interpolation it plays negative role. Consequently, the luminous intensity of each LS at one point has several values, which makes interpolation impossible.

In order to solve this problem, the angle $\Theta = 0°$ is replaced by the angle $\Delta\Theta/100$ in photometric tables and the angle $\Theta = 180°$ is replaced by $(180° - \Delta\Theta/100)$. As a result, different values of luminous intensity in the neighbourhood of $\Theta = 0°$ and 180° correspond to different (though closely positioned) points of space.

Second, while the initial mesh (1) in the own coordinate system of a specific LS covers the whole region [0°; 180°]×[0°; 360°], the non-regular mesh in the common system obtained by means of transformations (7) "backs out" of its edges (Fig. 3).

It is necessary to apply extrapolation instead of interpolation in regions not covered by the mesh, which leads to high errors.

In order to solve this problem, the initial (regular) mesh is widened and periodicity of functions $I_j(\Theta, \Phi)$ by both variables and their parity relative to Θ are used. For instance, instead of the range [0°, 360°], the change range of angle Φ is considered: from $10\Delta\Phi$ to $360° + 10\Delta\Phi$. Experience has proven that the described widening is sufficient for full covering of the region [0°; 180°]×[0°; 360°] by the non-regular mesh in the common coordinate system.

5. EXPERIMENT DESCRIPTION

The actual data required to perform comparative analysis of the methods described above was obtained in the course of the goniophotometer experiment. The LED based LSs used in the experiment are presented in Fig. 4. The first one (LS1) is an oblique luminaire on a basis of a *Feron 3602 LB-24 MR16* LED lamp and the second one (LS2) is a LED lamp for accent lighting with axially symmetric light distribution with power comparable to that of LS1.

Luminous intensity was measured in standard conditions by means of a *GO2000A* goniophotometer set containing: *GO2000A* goniometer (range of rotation in horizontal and vertical planes: ±180°, accuracy of rotation angle setting: 0.1°); *ID-1000* photometer based on a silicone photodiode adjusted for the function $V(\lambda)$, accuracy class L; *DPS1060* power supply unit.

All photometric data used in the further calculations is the arithmetic mean of the results of 5 measurements.

Photometry was performed in the system (C, γ). Measurement interval was 5° for plane C and 1° for plane γ. The experimental PBs of each LS are presented in Fig. 5.

For measurement of total light distribution of the said sources (Fig. 6), LS1 was installed so that its geometrical axis was parallel to the axis of photometry and orientation of LS2 was defined by the sequence of rotations of its geometrical axis: by 46° around the axis *Ox* and by 190° around the axis *Oz*. The common coordinate system was affixed to LS1.

5. COMPARISON OF CALCULATION METHODS

The comparison criteria of the calculation methods included simplicity of algorithm implementation, speed of their work, and accuracy.

Both methods were implemented by means of *Wolfram Mathematica*. The input photometric data was imported from the *XLS* file by means of embedded functions of this mathematical software. Both DFT and piecewise-linear interpolation in a non-regular mesh are standard functions of this software and *Mathematica* uses the Delaunay triangulation for this interpolation. With consideration of these circumstances, complexity of programming using this software as basically the same for both methods.

The same may be said about their processing speed. The calculations were performed using a laptop with 2,400GHz *Intel Core i7–4500U Haswell* CPU with 6Gb of RAM and powered by *Win 8.1×64*. In both cases, the computation required (15–20) minutes and the total light distribution values calculated using both methods were rather close to those experimentally discovered (Fig. 7).

The accuracy criterion was relative error of calculation of the values of total luminous intensity of two LSs as compared to its experimental (measured) values I_{pq} at points (9). Comparison of the calculated and experimental data was performed only within the region $I_{pq} \geq I_{max}/2$ where I_{max} is the largest measured value of total luminous intensity. Such limitation allows us to ignore the regions which are not illuminated by the LSs [3]. On the other hand, it is these regions not interesting from technical point of view where relative error may drastically increase due to smallness of the measured values.

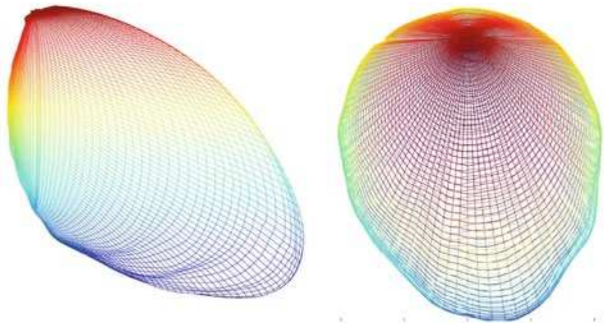

Fig. 6. Total photometric bodies acquired experimentally

In the selected region, maximum value of error was less than 4 % for trigonometric interpolation and about 6.5 % for piecewise-linear interpolation.

6. CONCLUSION

The article proposes and analyses two methods of calculation of total luminous intensity distribution of several LSs with different spatial orientation with their positions being conventionally the same. The first one is related to trigonometric interpolation of photometric data, the second one is related to its piecewise-linear interpolation.

During the numerical experiment with use of real photometric data, trigonometric interpolation

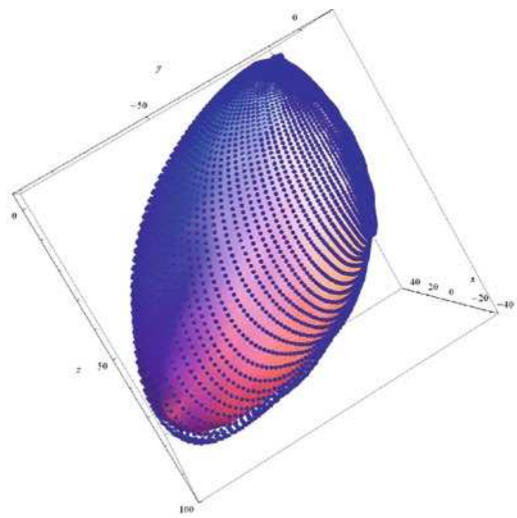

Fig. 7. Total luminous intensity distribution obtained by means of trigonometric interpolation (the points stand for experimental data, the solid surface stands for the results of calculation)

proved to be more accurate. With that, its error is partially related to exclusion of small summands out of expressions (6). That is why it may be also reduced by using a larger number of summands; however, this would impair processing speed of the method.

With the defined data set (2), there are no reserves for increase of accuracy of the method of piecewise-linear interpolation.

It is worth noting that PB's described by means of formulae of the form (3) and defined by means of trigonometric interpolation may be of inherent value. Being defined once, they may be used for different calculations, e.g. for transformation of photometric systems.

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