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# A SIMPLE TECHNIQUE TO DETERMINE SNOW PROPERTIES USING LIGHT REFLECTANCE MEASUREMENTS

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*All models are wrong but some are useful*
*George E.P. Box*

*Devoted to memory of G.V. Rozenberg (1914–1982)*

**ABSTRACT**

In this paper we review theoretical foundations of reflectance spectroscopy of snow. Simple approximate equations are presented, which can be used to calculate snow absorption/extinction coefficients and also snow reflectance. The equations derived could also be used to solve the inverse radiation transfer problem. The technique can be applied to other types of turbid media with large weakly absorbing particles. It has potential for the interpretation both: ground-based and airborne, or satellite, measurements of light reflected from cryosphere of our planet, and also has potential for applications to planetary imaging spectroscopy in general.

**Keywords**: reflectance spectroscopy, radiative transfer, light scattering, light absorption, light reflectance, inverse problem, snow, snow pollution, snow grain size, cryosphere, remote sensing

## 1. INTRODUCTION

Reflectance spectroscopy [1] is a standard tool for the characterization of turbid media such as soil, blood, paints, leaves, etc. It is based on the measurements of the light reflectance from a given sample as a function of the wavelength $\lambda$ and is a rapidly growing science that can be used to derive significant information about various materials with little or no sample preparation. The reflectance spectroscopy is much simpler as compared to the transmittance spectroscopy for the case of turbid media such as, say, rocks and minerals. It can be used to monitor various surfaces using airborne and satellite measurements. Also ground-based and ship-borne measurement systems based on the reflectance spectroscopy principles are often used.

In this paper a simple approach to the characterization of weakly absorbing strongly light scattering turbid media is reviewed and applied to the case of snow characterization (snow grain size, concentration, and pollution absorption coefficient/type). In the next section we shortly describe snow microphysics. Afterwards local optical characteristics of snow are discussed. Section 4 is aimed at studies of radiative transfer processes in snow. Then we solve the inverse radiative transfer problem for natural snow (section 5) and summarize the results.

## 2. SNOW MICROPHYSICS

Snow grains originate from precipitating crystalline clouds. Therefore, as in ice clouds, grains have very diverse shapes. Snow metamorphism is driven by gradients in vapour pressure, which in turn are driven by temperature gradients. Small temperature gradients (less than 10 degree per meter) result in small vapour pressure gradients and slow grain growth within the snowpack. The result is the formation of rounded snow grains that tend to be

0.1 to 0.2 mm in diameter. One explanation for the formation of rounded snow grains is that vapour diffusion within the snowpack causes a loss of mass from points on individual snow grains to gains in mass in hollows. Depth hoar forms in areas of a snowpack where temperature gradients are greater than 10ºC per meter. The process of forming these snow grains has a number of synonyms, including the historical term of temperature gradient metamorphism (TG metamorphism), constructive metamorphism, and kinetic growth. The large temperature gradient induces a large gradient in vapour pressure, such that water vapour moves from warmer areas of the snowpack with relatively higher vapour pressures across pore spaces to colder areas of the snowpack with lower vapour pressure. These conditions produce angular or faceted grains, which may later develop steps and striations on their surface, resulting in cup-shaped crystals with a hollow centre that generally range in size from (3–8) mm. Under very favourable conditions, individual grains can be larger than 15 mm.

The ice crystals in snow can be solid, hollow, broken, abraded, partly melts, rounded or angular. The surface of crystals can be rimed, stepped or striated. The rounded facets can be present as well. The crystals can be bounded, unbounded, clustered, or arranged in columns.

It is easy to derive the average radius of water cloud droplets characterized by the droplet size distribution

$$f(a):\ \bar{a} = \int_0^\infty af(a)da. \quad (1)$$

Here $a$ is the radius of droplets. The same procedure is not easy for snow because particles of different shapes and morphology present in snow. The practical way of the solution of this problem is the measurement of the largest snow grain size (diameter for spheres). The snow grain size as determined from optical measurements is close to the Sauter mean diameter $d_{ef}$ defined as the diameter of the sphere having the same volume/surface area (*V/S*) ratio as an ice crystal of interest. This is due to the fact that clear snow spectral reflectance $R(\lambda)$ is determined mostly by the snow spectral single scattering albedo, which is primarily the function of $d_{ef}$. The value of $d_{ef}$ for snow, having different shapes and sizes, is defined as follows:

$$d_{ef} = \frac{6\int_0^\infty Vf(V)dV}{\int_0^\infty Sf(S)dS}. \quad (2)$$

Here $f(V)$ is the volume distribution function and $f(S)$ is the surface area distribution function. The integral in the nominator gives the average volume of grains and the dominator is the average surface area of the grains. It is clear that for the mono-dispersed spheres $d_{ef}=2a$ and it follows for the spherical poly-dispersions:

$$d_{ef} = \frac{2\int_0^\infty a^3 f(a)da}{\int_0^\infty a^2 f(a)da}. \quad (3)$$

Therefore, the ratio of the third to the second moment of the size distribution is involved in the definition of the Sauter mean diameter. The snow specific surface area (SSA) defined as the total surface area per unit of mass ($m^2/kg$) can be also related to the Sauter diameter:

$$SSA = \frac{6}{\rho d_{ef}}. \quad (4)$$

Here $\rho = 0.9167 g/cm^3$ is the density of ice. Because spectral reflectance depends on diameter of snow grains, it can be used to derive both snow grain size and snow specific surface area, which are important parameters for many applications including snow pollution and climate research.

The snow water equivalent (SWE) is defined as the depth of *water* that would theoretically result if you melted the entire snowpack instantaneously. It can be estimated as follows:

$$\text{SWE}=\rho_s l. \quad (5)$$

Here $l$ is the depth of the snowpack and $\rho_s$ is the snow density ((0.1–0.8) $g/cm^3$ depending on snow type). The SSA and SWE are characteristics of snowpack needed for numerous applications. Therefore, they are measured routinely in field and also derived using remote sensing techniques. The value of $l$ can be derived with airborne laser systems us-

**Table 1. The Concentration of Soot in Snow in Different Areas [2]**

| Location | Black carbon concentration (ng/g) |
|---|---|
| South Pole | 0.1–0.3 |
| Summit, Greenland | 1–30 |
| Spitzbergen | 7–52 |
| Barrow | 7–60 |
| Alert, N. Canada | 0–127 |
| French Alps | 4–826 |
| Urban Michigan | 17–5700 |

ing as a reference terrain not covered by snow (say, in summer).

Natural snow contains various types of pollutants. They originate from atmosphere (e.g., aerosol particles such as dust, soot, etc.), are of biological nature (e.g., algae) or from neighbouring objects such as trees (litter, tree brunches, etc.), rocks, and neighbouring bare soil. The extreme forms of snow pollution are well documented. Although the pristine snow fields are more common. Typical concentrations of soot in snow in different areas are given in Table 1. One can see that the more pure snow is in Antarctica. This is due to large distance to pollution sources. Nevertheless, the snow pollution due to biological material occurs in Antarctica as well.

One important applied problem is the determination of concentration/type of pollutants in snow. The concentration of Table 1 pollutants can be assessed studying level of snow darkening in the visible. The type of pollutants (algae, soot, dust) can be estimated from the spectral shape of measured snow reflectance.

## 3. LOCAL OPTICAL CHARACTERISTICS OF SNOW

### 3.1. Light Extinction in Snow

Extinction coefficient $\sigma_{ext}$ is the basic quantity for any turbid medium. It shows how fast the direct light beam attenuates in the medium due to combined scattering and absorption processes. In particular, it follows:

$$I = I_0 \exp(-\sigma_{ext} l), \tag{6}$$

where $l$ is the *geometrical thickness* of a snow sample, $I$ is the intensity of transmitted light and $I_0$ is the intensity of incident light. The measurements of spectral extinction coefficients of homogeneous media are quite simple and can be derived from equation given above. This task is not so easy for snow samples because one must remove the contribution of multiply scattering light into the detector. The extinction coefficient is defined via the extinction cross section $C_{ext}$ using the following equation:

$$\sigma_{ext} = N < C_{ext} >, \tag{7}$$

where $N$ is the number of snow grains in unit volume. It is known that for large scatters, the value of $C_{ext}$ is equal to the double of the cross sectional area $A$ (perpendicular to the incident beam) of the particle. Therefore, one derives:

$$\sigma_{ext} = 2AN. \tag{8}$$

The value of $N$ can be expressed via the volume concentration of particles $c$ and the average volume of particles:

$$N = \frac{c}{\langle V \rangle}. \tag{9}$$

Then it follows:

$$\sigma_{ext} = \frac{c}{p}, \tag{10}$$

where the parameter $p$ is given by the following equation:

$$p = \frac{\langle V \rangle}{2\langle A \rangle}. \tag{11}$$

In case of convex particles, the average cross section (at random orientation) coincides with the average surface area of particles multiplied by 4 [3]. Therefore, one derives:

$$p = \frac{2\langle V \rangle}{\langle S \rangle}. \tag{12}$$

Many particles in snow can have concave forms. Then Eq. (12) must be modified:

$$p = \upsilon \langle V \rangle / \langle S \rangle,$$

where the parameter $\upsilon$ depends on the type of snow.

Let us introduce the average diameter of particles:

$$d_{ef} = \frac{6\langle V \rangle}{\langle S \rangle}. \quad (13)$$

Then it follows: $p = d_{ef} / 3$ and, therefore,

$$\sigma_{ext} = \frac{3c}{d_{ef}}, \quad (14)$$

where we have assumed that grains have convex shapes (say, rounded ice particles). Taking into account that $c$ is often close to 1/3 for snow, one derives: $\sigma_{ext} \approx 1/d_{ef}$. Therefore, the light extinction length $L_{ext} = 1/\sigma_{ext}$ in snow is approximately equal to the effective snow grain diameter.

### 3.2. Scattering of Light in Snow

Snow reflective properties are determined by light scattering and absorption processes inside snow cover. In assumption that close packing effects can be ignored one can use the physical optics approximation for the calculation of the angular scattering pattern by a single ice grain. In this approximation one may assume that the phase function of an ice grain can be presented as a sum of two parts: diffraction part and geometrical optics part. The result for the phase function can be written in the following way:

$$p(\theta) = \frac{C_{sca,d}\, p_{sca,d}(\theta) + C_{sca,g}\, p_{sca,go}(\theta)}{C_{sca,d} + C_{sca,go}}, \quad (15)$$

where $C_{sca,d}$ is the diffraction part of the scattering cross section, $C_{sca,go}$ is the geometrical optics part of the scattering cross section $C_{sca}$, $p_{sca,d}(\theta)$ is the diffraction contribution to the total phase function, and $p_{sca,go}(\theta)$ is the geometrical optics contribution to the total phase function. The phase function is normalized as follows:

$$\frac{1}{2}\int_0^{\pi} p(\theta)\sin\theta d\theta = 1, \quad (16)$$

where $\theta$ is the scattering angle equal to zero in forward scattering direction and 180 degrees in the backward direction. In case of equal probability for light scattering by a scatterer at any scattering angle, one can easily derive: $p = 1$ (isotropic scattering). For large ice grains, there is a pronounced asymmetry in light scattering pattern: most of light scatterers in the forward scattering region. Asymmetry of phase function is described by the asymmetry parameter

$$g = \frac{1}{2}\int_0^{\pi} p(\theta)\cos\theta\sin\theta d\theta, \quad (17)$$

which is equal to the average cosine of scattering angle. One case also introduce the parameter symmetry $s=1-g$. Clearly, the value of $s$ is equal to 1 for the case of isotropic scattering ($g=0$). It follows from Eqs. (15), (17):

$$g = \frac{C_{sca,d}\, g_d + C_{sca,g}\, g_g}{C_{sca,d} + C_{sca,g}}. \quad (18)$$

In case of large non absorbing grains one can derive [3]: $C_{sca,d} = C_{sca,go}$ and, therefore,

$$g = \frac{1 + g_{go}}{2}, \quad (19)$$

where we accounted for the fact that the diffraction occurs in the forward scattering direction and, therefore, $g_d = 1$. The measurements of $g$ in ice clouds composed of irregular shaped particles [4] give values of $g$ close to 0.75, and, therefore, $g_{go}$=1/2. The value of $g_{go}$ depends on the shape of particles and also on the refractive index, being larger for rounded scatterers and also for weakly refracting particles with the real part of refractive index close to 1. The snow phase functions are difficult to measure and, therefore, for modelling purposes it is assumed that they are close to phase functions of ice clouds composed of large irregular ice grains. Such phase functions are featureless and almost constant at the backward scattering hemisphere. They produce featureless snow reflectance patterns. In particular rainbows and glories seen in reflected light for water clouds are never observed for snow covers. The equations presented above are valid for a single snow grain. Therefore, the averaging procedure shall be applied to get the local snow optical properties. In particular, it follows for the snow phase function:

$$p_s(\theta) = \frac{< C_{sca,d}\, p_{sca,d}(\theta) > + < C_{sca,g}\, p_{sca,go}(\theta) >}{< C_{sca,d} > + < C_{sca,go} >}, \quad (20)$$

where angular brackets mean averaging with respect to the size of snow grains and also their shapes. The following expression can be derived for the snow asymmetry parameter:

$$g = \frac{< C_{sca,d} > + < C_{sca,g}\, g_{go} >}{< C_{sca,d} > + < C_{sca,go} >}, \quad (21)$$

where we have taking into account that the asymmetry parameter for the diffraction part is close to one. For non absorbing particles, $g_{go}$ does not depend on the size of particles and one can derive:

$$g = \frac{1 + < g_{go} >}{2}, \quad (22)$$

where angular brackets mean averaging with respect to the shape of particles we have taken into account that the geometrical optics and diffraction parts of average scattering cross sections coincide for nonspherical particles and have assumed that $< C_{sca,g}\, g_{go} > = < C_{sca,g} >< g_{go} >$.

The phase function of snow has not been measured *in situ* so far. This function is modelled assuming various shapes of crystals in the framework of geometrical optics (ray tracing). The application of geometrical optics is possible because ice grains are much larger as compared to the wavelength of the incident light. This simplifies the problem in great extent avoiding the use of Maxwell theory, which does not lead to the closed form solutions for the irregularly shaped particles. The parameterizations of the snow phase function useful for studies of radiation transport in crystalline clouds and snow has been developed in [5, 6].

### 3.3. Light Absorption in Snow

Snow grains not only scatter light but also some portion of light is absorbed by snow grains. The absorption processes can be neglected in the visible. However, they are of importance in the near infrared, where ice absorbs light with various degrees of strength depending on the actual wavelength. The absorption cross section $C_{abs}$ of a single ice grain can be presented in the following form:

$$C_{abs} = \frac{k}{\left|\vec{E}_0\right|^2} \int_V \varepsilon''(\vec{r}) \vec{E}(\vec{r}) \vec{E}^*(\vec{r})\, d^3\vec{r}. \quad (23)$$

Here, $k = \frac{2\pi}{\lambda}$ is the wave number, $V$ is the volume of an ice grain, $\varepsilon''(\vec{r}) = 2n\chi$ is the imaginary part of the relative dielectric permittivity of a particle, $m=n-i\chi$ is the complex refractive index of ice grain, $\vec{E}_0$ is the incident electric field, $\vec{E}(\vec{r})$ is the electric field inside the particle. Let us introduce the average normalized intensity of light inside the particle:

$$\Pi = \frac{1}{V} \int_V \frac{\left|\vec{E}(\vec{r})\right|^2}{\left|\vec{E}_0\right|^2} d^3\vec{r}. \quad (24)$$

Then it follows from Eq. (23) assuming that the particle is internally homogeneous:

$$C_{abs} = n\alpha \Pi V, \quad (25)$$

where the factor $\Pi$ depends on the size, shape, and complex refractive index of particles and $\alpha = 2k\chi$. It is clear that the value of $\Pi$ is close to one for weakly absorbing particles as $n \to 1$ because it follows in this case: $\vec{E}(\vec{r}) \approx \vec{E}_0(r)$. Therefore, one derives:

$$C_{abs} = \alpha V. \quad (26)$$

For large non absorbing particles with large differences $\Delta n = n - 1$, the value of $\Pi$ also does not depend on the size of particles [7]. This is also approximately true for absorbing particles, if $\chi / n << 1$, $\chi x << 1$ (valid for snow in the visible and near – infrared). Here $x = ka$ is the size parameter, $a$ is the characteristic size (radius for monodispersed spheres) of a scatterer. Therefore, Eq. (26) is modified:

$$C_{abs} = B\alpha V, \quad (27)$$

where $B$ depends on the shape of particles and real part of $n$ but not on the size of particles. The experimental measurements of the value of $B$ for the natural snow has been performed in [8]. It has been found that the average value of $B$ is 1.6 with some variation depending on the actual snow type. The value of $B$ for spherical ice grains is close to 1.25 [9]. Therefore, it follows that the use of spherical approximation will lead to underestimation of snow absorption and overestimation of snow reflectance.

The absorption coefficient:

$$\sigma_{abs} = N\langle C_{abs}\rangle, \quad (28)$$

can be presented, therefore, in the following form:

$$\sigma_{abs} = B\alpha c, \quad (29)$$

where $c$ is the volumetric concentration of snow grains equal to the ratio of snow $\rho_s$ and ice $\rho_i$ densities. This ratio is close to 1/3 for many types of snow. This means that the absorption coefficient of snow is approximately two times smaller as compared to that of bulk ice and has almost the same spectral behaviour as bulk ice in the visible and near infrared regions of the electromagnetic spectrum. The result presented above is valid only for the case of weakly absorbing snow grains. It must be modified in case of moderate and strongly absorbing particles (say, ice grains at 1.6 and 2.1 microns, where light absorption by large snow grains is outside weak absorption limit).

### 3.4. Local Optical Properties of Polluted Snow

Polluted snow is composed of ice grains and various pollutants (dust, soot, algae, etc.). The local optical properties can be found assuming external mixing rules for the extinction coefficient, absorption coefficient, and phase function:

$$\sigma_{ext} = \sigma_{ext,i} + \sigma_{ext,p} \equiv N_i\bar{C}_{ext,i} + \sum_{p=1}^{N} N_p\bar{C}_{ext,p}, \quad (30)$$

$$\sigma_{abs} = \sigma_{abs,i} + \sigma_{abs,p} \equiv N_i\bar{C}_{abs,i} + \sum_{p=1}^{N} N_p\bar{C}_{abs,p}, \quad (31)$$

$$p(\theta) = \frac{\bar{C}_{sca,i}\, p_{sca,i}(\theta) + \sum_{p=1}^{N} \bar{C}_{sca,p}\, p_{sca,p}(\theta)}{\bar{C}_{sca,i} + \sum_{p=1}^{N} \bar{C}_{sca,p}}, \quad (32)$$

where indices $i$, $p$ is signify the contribution of ice ($i$) and $N$ pollutants ($p$) and the scattering cross section:

$$C_{sca} = C_{ext} - C_{abs}. \quad (33)$$

In most of cases one needs to account for the presence of just one ($N=1$) pollutant (let's us say soot). Also in many cases scattering of light by pollutants is small with absorption processes predominated. Then one needs to account for the presence of pollutants just in calculation of absorption coefficient. Although such assumptions are often used in snow applied optics they may lead to biases in calculations because in reality pollutants can have large optical sizes and concentrations. Then one can not ignore scattering of light by pollutant particles anymore. Also some pollutants can be internally mixed [10].

## 4. RADIATIVE TRANSFER IN SNOW

### 4.1. Radiative Transfer Equation Approach

The snow radiative transfer characteristics are usually studied in the framework of scalar radiative transfer theory. Therefore, the well known radiative transfer equation (RTE) for the intensity of light filed $I$ given below is solved (for a given direction specified by a solid angle Ω) [11]:

$$\mu\frac{dI(\Omega)}{d\tau} = -I + \frac{\omega_0}{4\pi}\int_{\Omega} p(\Omega',\Omega)I(\Omega')d\Omega', \quad (33)$$

where we have assumed that snow can be presented as a horizontally homogeneous plane-parallel light scattering layer and effects of thermal emission can be ignored, which is certainly true in the visible and near IR regions of electromagnetic spectrum. Here, $\mu$ is the cosine of the viewing zenith angle counted from the normal to the snow layer, $\omega_0 = 1 - \sigma_{abs}/\sigma_{ext}$ is the single scattering albedo, and we have introduced the snow optical depth (SOD) $\tau = \sigma_{ext} l$.

For inhomogeneous layers, SOD is defined as an integral of the extinction coefficient via vertical coordinate. In reality, due to accumulation processes (say, several snowfalls and pollution deposition events) snow has a layered structure and assumption of a vertically homogeneous layer must be taken with precaution.

RTE can be solved using a number of numerical and approximate analytical techniques providing the dependence of the intensity of reflected, transmitted and internal light field $I$ on a number important parameters such as snow grain size, snow grain shape, snow density, type, concentration of pollutants, size distribution of various pollutants, snow thickness. The snow layer albedo and absorptance can be also calculated.

The influence of underlying surface reflectance (let's say for thin snow layers) can be also studied using appropriate boundary conditions.

It should be pointed out that RTE given above assumes that particles in a scattering layer are not oriented (random distribution of irregular shaped particles) and also they are not in contact and at large distances one from another. The second condition is actually violated for snow because volumetric concentration of ice grains is about 0.3. Therefore, the application of the standard for of RTE can lead to large errors. This is certainly true in the thermal infrared in microwave regions of the electromagnetic spectrum. However, experimental measurements of snow reflectance show that standard RTE can be applied in the visible and near infrared range of electromagnetic spectrum [12]. This is due to the fact that light scattering in snow occurs in geometrical optics domain because the grains are typically have the sizes (100–1000) times larger as compared to the wavelength of the incident light. Also the particles are irregularly shaped. Therefore, dense media effects are washed out.

### 4.2. Analytical Approximation of the Snow Spectral Reflectance

In applied research it is often desirable to have analytical equations relating the measured characteristics, let's say, snow layer spectral reflectivity, with snow microstructure parameters such as snow grain size and concentration of pollutants. This makes it possible to simplify the inverse problem solution. In this section we shall derive such an equation based on the statistical approach not directly relying on the RTE, which has limitations as far as dense media effects are of concern.

Let us consider the case of absorbing snow. The reflectance $R = \pi I / \mu_0 E_0$ ($\mu_0$ is the cosine of the incident zenith angle, $E_0$ is the incident light flux on the unit area perpendicular to the incident beam) can be presented in the following way:

$$R(\beta) = \sum_{n=1}^{\infty} a_n (1-\beta)^n, \tag{34}$$

where $\beta = 1-\omega_0$ is the probability of photon absorption (PPA) by unit volume of snow. In the case of non absorbing snow it follows:

$$R(0) = \sum_{n=1}^{\infty} a_n . \tag{35}$$

Therefore, one derives for $\Re = R(\beta)/R(0)$:

$$R = \frac{\sum_{n=1}^{\infty} a_n (1-\beta)^n}{\sum_{n=1}^{\infty} a_n}. \tag{36}$$

Expanding $(1-\beta)^n$ as $\beta \to 0$, we derive:

$$R \approx 1 - \beta < n > + \\ + \frac{\beta^2}{2} < n^2 > - \frac{\beta^3}{6} < n^3 > + ... \approx < \exp(-\beta n) >, \tag{37}$$

where

$$\langle n^k \rangle \equiv \sum_{n=1}^{\infty} f_n n^k, \ \langle \exp(-\beta n) \rangle \equiv \sum_{n=1}^{\infty} f_n \exp(-\beta n),$$

$$f_n = \frac{a_n}{\sum_{n=1}^{\infty} a_n}, \tag{38}$$

and we assumed that $n(n-1) \approx n^2$, $n(n-1)(n-2) \approx n^3$, … in our derivations. This is possible because $\beta$ is close to zero and the number of scattering events in snow is high in the visible and near infrared regions of the electromagnetic spectrum. For the same reason we have:

$$< \exp(-\beta n) > \approx \int_0^{\infty} f(n) \exp(-\beta n) dn. \tag{39}$$

This integral can be evaluated assuming the function $f(n)$. In particular, it follows from the random walk theory $[13]$ that that the probability of a particle (photon) appearing at a given place, time, and direction after *large* number of iterations can be presented as

$$f(n) = \sqrt{\frac{\eta}{\pi}} n^{-3/2} \exp\left\{-\frac{\eta}{n}\right\}, \quad (40)$$

where the parameter $\eta$ depends on the process studied. The substitution of Eq. (40) into Eq. (37), (39) gives:

$$R = \exp(-2\sqrt{\eta\beta}). \quad (41)$$

Therefore, we can write

$$R(\beta) = R_0 \exp(-\sqrt{s\beta}), \quad (42)$$

where $R_0 \equiv R(0)$, $s = 4\eta$. This equation shows how the spectral snow reflectance depends on the probability of photon absorption $\beta$. The parameter $s$ depends on the scattering and not on absorption processes and, therefore, one may assume that it does not depend on the wavelength for snow composed of large snow grains in contact. Eq. (42) is very general and can be applied to many types of light scattering media. It has been derived for the first time in $[14]$. In the next section we shall apply Eq. (42) for the interpretation of the experimentally measured snow spectral reflectance and also for the solution of the inverse radiative transfer problem for the case of a homogeneous semi-infinite snow layer.

The value of $s$ can be related to the asymmetry parameter *g* of ice grains using asymptotic results of RTE valid as $\beta \to 0$. Then it follows $[15]$:

$$R(\beta) = R_0 - yu(\mu_0)u(\mu), \quad (43)$$

where

$$y = 4\sqrt{\frac{\beta}{3(1-g)}} \quad (44)$$

and

$$u_0(\mu_0) = \frac{3}{4}(\mu_0 + \varphi(\mu_0)), \quad (45)$$

$$\varphi(\mu_0) = 2\int_0^1 \langle R_0(\mu_0, \mu)\rangle \mu^2 d\mu, \quad (46)$$

$$\langle R_0(\mu_0, \mu)\rangle = \frac{1}{2\pi}\int_0^1 R_0(\mu, \mu_0, \psi) d\psi. \quad (47)$$

One can show [16] that the following approximation holds:

$$u(\mu_0) = \frac{3}{7}[1 + 2\mu_0]. \quad (48)$$

Comparing Eqs. (43) and (42) (at small values of PPA), one derives:

$$s = \frac{16u^2(\mu_0)u^2(\mu)}{3(1-g)R_0^2(\mu_0, \mu, \psi)}. \quad (49)$$

## 5. THE APPROXIMATE SOLUTION OF THE INVERSE RADIATIVE TRANSFER PROBLEM

Equations presented above can be used to establish the analytical relationship between the snow spectral reflectance and diameter of ice grains. To simplify, we assume that there is just one type of pollutant in snow. Then it follows for PPA:

$$\beta = \frac{\sigma_{abs,i} + \sigma_{abs,p}}{\sigma_{ext,i} + \sigma_{ext,p}}, \quad (50)$$

where indices *i, p* are signify ice grains and pollutants, respectively. Under assumption, that extinction of light by pollutant is much smaller (see Table 1) as compared to that by ice grains, one derives (Eqs. (14), (29)):

$$\beta = \left[\frac{B\alpha(\lambda)}{3} + \frac{\sigma_{abs,p}(\lambda)}{3c_i}\right] d_{ef} \quad (51)$$

and, therefore,

$$R(\lambda) = R_0 \exp\left\{-\sqrt{\left[\alpha(\lambda) + F\sigma_{abs,p}(\lambda)\right]D}\right\}, \quad (52)$$

where

$$F = \frac{1}{Bc_i}, \quad (53)$$

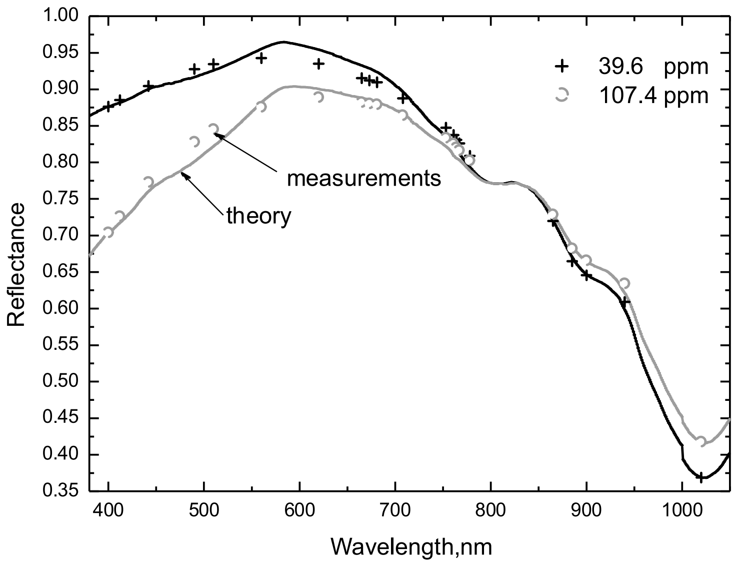


Fig. 1. The spectral reflectance of dust – polluted (with dust concentration 39.6 and 107.4 ppm) snow measured in European Alps as described in [17] and theoretical modelling according Eq. (57) with the bulk ice absorption [19] coefficient calculated using the spectral ice refractive index tabulated in. The derived values of $m$, $d_{ef}$ were 4.1, 2.5 mm for the case of weakly polluted and 6.4, 1.5 mm, respectively, for the case of strongly polluted snow. The derived absorption coefficient of pollutants at the wavelength 560 nm was 0.12 for the weakly polluted (39.6 ppm) snow and 0.31 for the strongly polluted (107.4 ppm) snow under assumption that the volumetric concentration of ice grains is 0.3. The observation has been performed in the nadir direction at the solar zenith angle equal to 52 degrees

$$D = \frac{B}{3} s d_{ef}. \tag{54}$$

In case of pure snow Eq. (52) is simplified:

$$R(\lambda) = R_0 \exp\left\{-\sqrt{\alpha(\lambda) D}\right\}. \tag{55}$$

Because the spectrum of the bulk ice absorption coefficient is a well known function $\alpha(\lambda)$ one can see that just two parameters $(D, R_0)$ are suffice to determine the clear snow reflectance spectrum in the visible and near infrared. These two parameters can be derived from measurements at two wavelengths 0.4 and 1.02 micrometers providing simultaneously snow grain size/SSA and $R_0$.

The absorption of pollutants can be parameterized as follows [17]:

$$\sigma_{abs,p}(\lambda) = c_p \kappa \tilde{\lambda}^{-m}, \tag{56}$$

where $c_p$ is the volumetric concentration of pollutants, $\kappa$ is the absorption coefficient of pollutants normalized at the wavelength $\lambda_0$ to the value of $c_p$, $\tilde{\lambda} = \lambda / \lambda_0$. Then it follows (see Eqs. (52), (56)):

$$R(\lambda) = R_0 \exp\left\{-\sqrt{\left[\alpha(\lambda) + \Phi \tilde{\lambda}^{-m}(\lambda)\right] D}\right\}, \tag{57}$$

where

$$\Phi = \frac{\tilde{c}_p \kappa}{B} \tag{58}$$

and

$$\tilde{c}_p = \frac{c_p}{c_i}. \tag{59}$$

Eq. (57) can be used to find the parameters $R_0$, $D$, $\Phi$, $m$ using, e.g., optimal estimation approach [18].

This enables the determination of the effective ice grain sizes ($d_{ef} = 3D / Bs$, see Eq. (54)) and also the spectral absorption coefficient of pollutants (at known values of concentration of ice grains, see Eqs. (56), (58), (59)).

Knowing the volumetric absorption coefficient of pollutants

$$\kappa = \frac{\bar{C}_{abs}(\lambda_0)}{\bar{V}_p}, \tag{60}$$

where $\bar{V}_p$ is the average volume of impurity particles, $\bar{C}_{abs}(\lambda_0)$ is the average absorption cross section of impurity particles, one can also find the normalized concentration of pollutants (say, soot) in snow (see Eqs. (58), (59)), which as an important applied problem. The four unknown parameters can be also found from Eq. (57) analytically from measurements at four wavelengths assuming that absorption of ice grains is negligible at the wavelengths $\lambda_1, \lambda_2$ in the visble and the absorptance of light by pollutants can be neglected in near – infrared, where bulk ice absorbs stronger (at the wavelengths $\lambda_3, \lambda_4$). The result is [17]

$$m = \frac{\ln(p_1/p_2)}{\ln(\lambda_2/\lambda_1)},\ \Phi = \frac{p_1\tilde{\lambda}_1^m}{D}, R_0 = $$
$$= R_3^{\varepsilon_1} R_4^{\varepsilon_2}, D = \alpha_4^{-1}\ln^2\left[\frac{R_4}{R_0}\right], \qquad (61)$$

where $R_1, R_2, R_3,$ and $R_4$ are the reflectances measured at four wavelengths, $p_k = \ln^2(R_k / R_0)$, and

$$\varepsilon_1 = (1-b)^{-1},\ \varepsilon_2 = 1-\varepsilon_1,\ b = \sqrt{\alpha_3/\alpha_4},$$

is the bulk ice absorption coefficient at the wavelength $\lambda_{3(4)}$.

The application of this approach to the measurements of polluted snow spectral reflectance is given in Fig.1. The parameters given by Eq. (61) have been found at the following wavelengths: 400, 560, 865, and 1020nm. A similar approach but applied to the measurements of both snow reflectance and albedo is presented in [17].

## 6. CONCLUSIONS

In this paper we have reviewed the theoretical foundations of snow reflectance spectroscopy. Although, an accurate treatment of the problem must be based on the integro-differential radiative transfer equation, we show that the approximations given by Eqs. (52), (55), (57) simplify the problem in great extent making it possible to perform snow spectroscopy using inexpensive instruments and simple software based either on analytical solution of the inverse problem (see Eq. (61)) or optimal estimation technique. Our approach is valid not only for snow but also for other types of materials with large weakly absorbing scatterers.

## 7. ACKNOWLEDGEMENTS

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